Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2024
MATHEMATICS PAPER CODE 65(B)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers These are in the nature of Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark(√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓)while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks ___________(example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past:- ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.
16	Any un assessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65(B))

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A **Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each**	
1.	If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then the value of x, for which A is an identity matrix, is (A) $\frac{\pi}{2}$ (B) π (C) 0 (D) $\frac{3\pi}{2}$	
Sol.	(C) 0	**1**
2.	If the matrix $A = \begin{bmatrix} 0 & 5 & -7 \\ a & 0 & 3 \\ b & -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the values of 'a' and 'b' are : (A) a = 5, b = 3 (B) a = 5, b = – 7 (C) a = – 5, b = – 7 (D) a = – 5, b = 7	
Sol.	(D) $a = -5, b = 7$	**1**
3.	If $\begin{vmatrix} x+2 & x-4 \\ x-2 & x+3 \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 1 & 3 \end{vmatrix}$, then the value of x is : (A) 1 (B) 2 (C) – 2 (D) – 1	

Sol.	(B) 2	**1**
4.	If $\begin{bmatrix} 8 & 14 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} X$, then matrix X is : (A) $\begin{bmatrix} 3 & 7 \\ 2 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 0 \\ -3 & 7 \end{bmatrix}$	
Sol.	(C) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix}$	**1**
5.	The value of k, for which f(x) = $\begin{cases} \dfrac{\sqrt{3}\cos x + \sin x}{3x + \dfrac{\pi}{2}}, & x \neq -\dfrac{\pi}{3} \\ k, & x = -\dfrac{\pi}{3} \end{cases}$ is continuous at $x = -\dfrac{\pi}{3}$, is : (A) $\dfrac{2}{3}$ (B) $-\dfrac{2}{3}$ (C) $\dfrac{3}{2}$ (D) 6	
Sol.	Since correct answer is not in the options given So, 1 mark may be given to all who attempted this question	**1**
6.	Let the vectors $\vec{a}$ and $\vec{b}$ be such that $\vert\vec{a}\vert = \sqrt{3}$ and $\vert\vec{b}\vert = \dfrac{2}{\sqrt{3}}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is : (A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{6}$ (D) $\dfrac{\pi}{2}$	
Sol.	(C) $\frac{\pi}{6}$	**1**

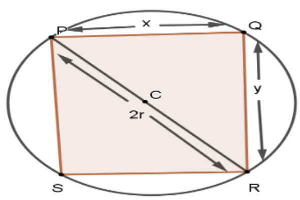
7.	If $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, then the projection of $(\vec{c} - \vec{b})$ along $\vec{a}$ is : (A) 15 (B) 5 (C) $\frac{2}{3}$ (D) 1	
Sol.	(B) 5	**1**
8.	The angle between the lines $\frac{x+1}{2} = \frac{2-y}{-5} = \frac{z}{4}$ and $\frac{x-3}{1} = \frac{y-7}{2} = \frac{5-z}{3}$ is : (A) $\frac{\pi}{4}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$	
Sol.	(B) $\frac{\pi}{2}$	**1**
9.	The Cartesian equations of a line are given as $6x - 2 = 3y + 1 = 2z - 2$ The direction ratios of the line are : (A) 2, – 1, 3 (B) 1, – 2, – 3 (C) 1, 2, 3 (D) 3, 1, 2	
Sol.	(C) 1, 2, 3	**1**
10.	The solution set of the inequation $2x + 3y < 6$ is : (A) open half-plane not containing origin (B) whole xy-plane except the points lying on the line $2x + 3y = 6$ (C) open half-plane containing origin (D) half-plane containing the origin and the points lying on the line $2x + 3y = 6$	
Sol.	(C) open half-plane containing origin	**1**

11.	The maximum value of the objective function $z = 3x + 5y$ subject to the constraints $x \geq 0, y \geq 0$ and $4x + 3y \leq 12$ is : (A) 15 (B) 29 (C) 9 (D) 20	
Sol.	(D) 20	**1**
12.	If the points A(3, – 2), B(k, 2) and C(8, 8) are collinear, then the value of k is : (A) 2 (B) – 3 (C) 5 (D) – 4	
Sol.	(C) 5	**1**
13.	If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$ is equal to : (A) $\frac{3}{2}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{3}{2}$	
Sol.	(D) $-\frac{3}{2}$	**1**
14.	$\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ is equal to : (A) $\cot x + \tan x + c$ (B) $-\cot x + \tan x + c$ (C) $\cot x - \tan x + c$ (D) $-\cot x - \tan x + c$	
Sol.	(D) $-\cot x - \tan x + c$	**1**

15.	The solution of the differential equation $\frac{dy}{dx} = 1 - x + y - xy$ is : (A) $\log\|1+y\| = x - \frac{x^2}{2} + c$ (B) $\log\|1+y\| = -x + \frac{x^2}{2} + c$ (C) $e^y = x - \frac{x^2}{2} + c$ (D) $e^{(1+y)} = -x + \frac{x^2}{2} + c$	
Sol.	(A) $\log\|1+y\| = x - \frac{x^2}{2} + c$	**1**
16.	The degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^5 = 0$ is : (A) 2 (B) 3 (C) 4 (D) 5	
Sol.	(B) 3	**1**
17.	The integrating factor of the differential equation $\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x$ is : (A) $e^{\sec x}$ (B) $\sec x + \tan x$ (C) $\sec x$ (D) $\cos x$	
Sol.	(C) sec x	**1**
18.	The probabilities of A, B and C solving a problem are $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{6}$ respectively. The probability that the problem is solved, is : (A) $\frac{4}{9}$ (B) $\frac{5}{9}$ (C) $\frac{1}{90}$ (D) $\frac{1}{3}$	

Sol.	(B) $\frac{5}{9}$	**1**
	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	*Assertion (A) :* $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$ *Reason (R) :* $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
20.	*Assertion (A) :* If the side of a square is increasing at the rate of 0·2 cm/s, then the rate of increase of its perimeter is 0·8 cm/s. *Reason (R) :* Perimeter of a square = 4 (side).	
Sol.	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	**1**
	SECTION B **This section comprises of Very Short Answer (VSA) type questions of 2 marks each.**	
21(a).	Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.	

Sol.	$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ $= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$ $= \frac{3\pi}{4}$	**1½** **½**
	OR	
21(b).	Find the domain of the function $y = \cos^{-1}(x^2 - 4)$.	
Sol.	$-1 \leq x^2 - 4 \leq 1$ $3 \leq x^2 \leq 5$ $\Rightarrow$ domain $= [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$	**1** **1**
22(a).	Differentiate $\cot^{-1}(\sqrt{1+x^2} + x)$ w.r.t. x.	
Sol.	$y = \cot^{-1}(\sqrt{1+x^2} + x)$, (Put $x = \cot\theta$) $= \cot^{-1}(\operatorname{cosec}\theta + \cot\theta) = \cot^{-1}\left(\frac{1+\cos\theta}{\sin\theta}\right)$ $= \cot^{-1}(\cot\frac{\theta}{2}) = \frac{1}{2}\cot^{-1}x$ $\frac{dy}{dx} = -\frac{1}{2(1+x^2)}$	½ ½ ½ ½
22(b).	If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.	
Sol.	$(\cos x)^y = (\cos y)^x$ $\Rightarrow y \log\cos x = x \log\cos y$ Differentiating, $\frac{dy}{dx}\log\cos x + y\,(-\tan x) = \log\cos y + x\,(-\tan y)\,\frac{dy}{dx}$	½ **1**

	$\frac{dy}{dx} = \frac{y \tan x + \log \cos y}{x \tan y + \log \cos x}$	½
23.	Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is (a) strictly increasing (b) strictly decreasing.	
Sol.	$f'(x) = -6 - 4x$ $f'(x) = 0$ gives $x = -\frac{3}{2}$ Intervals are $(-\infty, -\frac{3}{2})$ and $(-\frac{3}{2}, \infty)$ $f'(x) > 0$ for $x < -\frac{3}{2}$, f is strictly increasing in $(-\infty, -\frac{3}{2})$ *or* $(-\infty, -\frac{3}{2}]$ $f'(x) < 0$ for $x > -\frac{3}{2}$, f is strictly decreasing in $(-\frac{3}{2}, \infty)$ *or* $[-\frac{3}{2}, \infty)$	½ ½ ½ ½
24.	Show that of all rectangles inscribed in a given circle, the square has the maximum area.	
Sol.	Let the rectangle with sides x and y has maximum area $\therefore A = x . y$ where $x^2 + y^2 = 4r^2$, r being the radius  $\therefore A = x\sqrt{4r^2 - x^2}$ or $Z = x^2(4r^2 - x^2) = 4r^2x^2 - x^4$, where $Z = A^2$ $\frac{dZ}{dx} = 8r^2x - 4x^3$, $\frac{dZ}{dx} = 0 \Rightarrow x = \sqrt{2}r$ $\therefore y = \sqrt{4r^2 - 2r^2} = \sqrt{2}r$ $\frac{d^2Z}{dx^2} = 8r^2 - 12x^2 < 0$ for $x^2 = 2r$	 ½ ½ ½

	$\therefore x = y = \sqrt{2}r$ Therefore, of all the rectangles inscribed in a given circle, the square has the maximum area.	½
25.	Find : $$\int \text{cosec}^3\ (3x+1)\cot(3x+1)\ dx$$	
Sol.	$I = \int \text{cosec}^3(3x+1)\cot(3x+1)\,dx$ Put $\text{cosec}\,(3x+1) = t$, $-\text{cosec}\,(3x+1)\cot(3x+1)\,dx = \frac{1}{3}dt$ $I = -\frac{1}{3}\int t^2 dt = -\frac{1}{9}t^3 + c$ $= -\frac{1}{9}\text{cosec}^3(3x+1) + c$	**1** ½ ½
	SECTION C **This section comprises of Short Answer (SA) type questions of 3 marks each.**	
26.	If $x = a\cos\theta$ and $y = b\sin\theta$, then prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$.	
Sol.	$\frac{dx}{d\theta} = -\ a\sin\theta, \frac{dy}{d\theta} = \ b\cos\theta \implies \frac{dy}{dx} = -\ \frac{b}{a}\ \cot\theta$ $\frac{d^2y}{dx^2} = -\frac{b}{a}(-\text{cosec}^2\theta).\frac{d\theta}{dx} = \frac{b}{a}\text{cosec}^2\theta\left(-\ \frac{1}{a\ \sin\ \theta}\right) = \frac{-b}{a^2\sin^3\theta}$ $= -\frac{b^4}{a^2y^3}$	**1½** **1** ½
27.	Find : $$\int \frac{2x}{(x^2+1)(x^2+3)}\,dx$$	

Sol.	$I = \int \frac{2x}{(x^2+1)(x^2+3)}\, dx$, (Put $x^2 = t, 2xdx = dt$) $= \int \left(\frac{\frac{1}{2}}{t+1} - \frac{\frac{1}{2}}{t+3} \right) dt$ $= \frac{1}{2}[\log(t+1) - \log(t+3)] + c$ $= \frac{1}{2}[\log(x^2+1) - \log(x^2+3)] + c$ or $\frac{1}{2}\log\left\|\frac{x^2+1}{x^2+3}\right\| + c$	½ **1** **1** ½
28(a).	Evaluate : $\int_{-6}^{6} \|x+2\| dx$	
Sol.	$I = \int_{-6}^{6} \|x+2\|\, dx = \int_{-6}^{-2} \|x+2\|\, dx + \int_{-2}^{6} \|x+2\|\, dx$ $= \int_{-6}^{-2} -(x+2)\, dx + \int_{-2}^{6}(x+2)\, dx$ $= -\left[\frac{(x+2)^2}{2}\right]_{-6}^{-2} + \left[\frac{(x+2)^2}{2}\right]_{-2}^{6}$ $= -(0-8) + (32) = 40$	**1** ½ **1** ½
28(b).	Find : $\int \left(\frac{4-x}{x^5}\right) e^x\, dx$	
Sol.	$I = \int \left(\frac{4-x}{x^5}\right) e^x dx$ $= \int (4x^{-5} - x^{-4}) e^x\, dx$ (if $f(x) = -x^{-4}$, then $f'(x) = 4x^{-5}$) $I = -x^{-4}e^x + c$ or $-\frac{e^x}{x^4} + c$	 **1½** **1½**

29(a).	Find the particular solution of the differential equation $2xy\,\frac{dy}{dx} = x^2 + 3y^2$, given that $y(1) = 0$.	
Sol.	$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy} = \frac{1 + 3\left(\frac{y}{x}\right)^2}{2\,\frac{y}{x}}$ Put $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ $v + x\frac{dv}{dx} = \frac{1+3v^2}{2v} \Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v}$ $\Rightarrow \int \frac{2v}{1+v^2}dv = \int \frac{dx}{x} \Rightarrow \log(1+v^2) = \log x + \log c$ $\Rightarrow 1 + \left(\frac{y}{x}\right)^2 = cx, x = 1, y = 0 \Rightarrow c = 1$ $\Rightarrow x^2 + y^2 = x^3$ is the required solution	½ **1** **1** ½
29(b).	Solve the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$.	
Sol.	$\frac{dy}{dx} + 2y\tan x = \sin x$ I.F. $= e^{\int 2\tan x dx} = e^{2\log\sec x} = \sec^2 x$ $\therefore y\sec^2 x = \int \sin x \sec^2 x\, dx = \int \sec x \tan x\, dx$ $= \sec x + c$ $y = 0, x = \frac{\pi}{3} \Rightarrow 0 = 2 + c \Rightarrow c = -2$ $\Rightarrow y\sec^2 x = \sec x - 2$ Or $y = \cos x - 2\cos^2 x$	**1** **1½** ½

30.	The corner points of the feasible region determined by the system of linear constraints are A(0, 40), B(20, 40), C(60, 20) and D(60, 0). The objective function of the L.P.P. is z = 4x + 3y. Find the point of the feasible region at which the value of objective function is maximum and the point at which the value is minimum. Hence, find the maximum and the minimum values.	
Sol.	$Z = 4x + 3y$ Z_A = 120 Z_B = 200 Z_C = 300 Z_D = 240 Max Z = 300 at point C Min Z = 120 at point A	**2** **1**
31(a).	A card is randomly drawn from a well-shuffled pack of 52 playing cards. Events A and B are defined as under : A : Getting a card of diamond B : Getting a queen Determine whether the events A and B are independent or not.	
Sol.	$P(A) = P(\text{diamond}) = \frac{13}{52} = \frac{1}{4}$ $P(B) = P(\text{queen}) = \frac{4}{52} = \frac{1}{13}$ $P(A \cap B) = P(\text{queen of diamond}) = \frac{1}{52}$ Since $P(A).P(B) = \frac{1}{52} = P(A \cap B)$ $\Rightarrow$ A and B are independent events.	½ ½ ½ **1** ½
	OR	

31(b).	Find the probability distribution of the number of doublets in three throws of a pair of dice.	
Sol.	$p = P(\text{doublet}) = \frac{1}{6}, \quad q = 1 - \frac{1}{6} = \frac{5}{6}$ Let X denote the number of doublets. X can take values 0, 1, 2 or 3	**1** **1**
	See table below	**1**
	SECTION D **This section comprises of Long Answer (LA) type questions of 5 marks each.**	
32(a).	Let $A = \{x \mid x \in Z, 0 \le x \le 12\}$. Show that the relation $R = \{(a, b) : a, b \in A, (a-b) \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of elements related to 2.	

X	0	1	2	3
P(X)	$\left(\frac{5}{6}\right)^3 = \frac{125}{216}$	$3 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^2 = \frac{75}{216}$	$3 \times \left(\frac{1}{6}\right)^2 \times \frac{5}{6} = \frac{15}{216}$	$\left(\frac{1}{6}\right)^3 = \frac{1}{216}$

$R = \{(a, b) : a, b \in A, (a - b) \text{ is divisible by } 4\}$

(i) For every $a \in A$ we have $a - a = 0$ which is divisible by 4
$\Longrightarrow (a, a) \in R$ for all $a \in A$ Hence R is reflexive — **1**

(ii) Let $(a, b) \in R$, $a, b \in A$
$\Longrightarrow (a - b)$ is divisible by 4 $\Longrightarrow (b - a)$ is also divisible by 4
$\therefore (b, a) \in R$ Hence R is symmetric — **1**

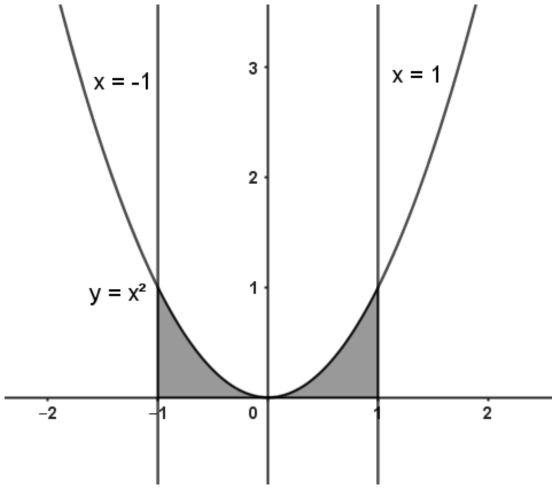
(iii) Let $(a, b) \in R$ and $(b, c) \in R$, $\because a, b, c \in A$
Then $(a - b)$ and $(b - c)$ are divisible by 4
$\Longrightarrow a - c = (a - b) + (b - c)$ which is divisible by 4
$\therefore (a, c) \in R$ Hence R is transitive — **2**

R is an equivalence relation

$[2] = \{2, 6, 10\}$ — **1**

OR

32(b).	Let $A = \mathbb{R} - \{4\}$ and $B = \mathbb{R} - \{1\}$ and let function $f : A \to B$ be defined as $f(x) = \frac{x-3}{x-4}$ for $\forall\, x \in A$. Show that f is one-one and onto.	
Sol.	$f(x) = \frac{x-3}{x-4}$ Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2)$ $\therefore \frac{x_1-3}{x_1-4} = \frac{x_2-3}{x_2-4}$ $\Rightarrow x_1 = x_2$ $\Rightarrow$ f is one-one Let $y = \frac{x-3}{x-4} \in \mathbb{R} - \{4\}$ Then $xy - 4y = x - 3$ $\Rightarrow x = \frac{4y-3}{y-1}$ Range f = B = Codomain f $\Rightarrow$ f is onto	**2½** **2½**
33.	Using matrices, solve the following system of linear equations : $3x + 4y + 2z = 8$; $2y - 3z = 3$; $x - 2y + 6z = -2$	
Sol.	$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$ or $AX = B$ $\|A\| = 2 \neq 0$ $\text{adj}(A) = \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}$ $\Rightarrow A^{-1} = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}$ $\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ $\Rightarrow x = -2, y = 3, z = 1$	½ **1** **2** **1** ½

34.	Using integration, find the area of the region bounded by the curve $y = x^2$, $x = -1$, $x = 1$ and the x-axis.	
Sol.	$y = x^2,\ x = 1, x = 0$ and $x = -1$  Required area $= 2\int_0^1 x^2 dx$ $= 2\left[\frac{\mathbf{x^3}}{\mathbf{3}}\right]_{\mathbf{0}}^{\mathbf{1}}$ $= 2\left(\frac{1}{3}\right) = \frac{2}{3}$	**2** **2** **1**
35(a).	Write the vector equations of the following lines and hence find the shortest distance between them : $\frac{x+1}{2} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$	
Sol.	Vector equations are $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} - 6\hat{j} + \hat{k})$ and $\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \mu(\hat{i} - 2\hat{j} + \hat{k})$ $\vec{a_2} - \vec{a_1} = (4\hat{i} + 6\hat{j} + 8\hat{k})$	**1** **1** ½

	$\overrightarrow{b_1}\times\overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i}-\hat{j}+2\hat{k}$ $SD = \frac{\lvert(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})\rvert}{\lvert\overrightarrow{b_1}\times\overrightarrow{b_2}\rvert}$ $= \frac{\lvert-16-6+16\rvert}{\sqrt{16+1+4}} = \frac{6}{\sqrt{21}}$	**1½** **1**
	OR	
35(b).	Find the length and the coordinates of the foot of the perpendicular drawn from the point P(5, 9, 3) to the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$. Also, find the coordinates of the image of the point P in the given line.	
Sol.	Any point on the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ is $Q(2\lambda+1, 3\lambda+2, 4\lambda+3)$ DRs of $\overrightarrow{PQ}$ are $(2\lambda-4, 3\lambda-7, 4\lambda)$ $\overrightarrow{PQ}\perp$ given line $\Longrightarrow 2(2\lambda-4)+3(3\lambda-7)+4(4\lambda)=0$ $\Longrightarrow \lambda = 1$ ∴ Point Q is (3,5,7) $PQ = \sqrt{4+16+16} = 6$ coordinates of image P' are (1,1,11)	**1** **1** **1** **1** **½** **½**
	SECTION E **This section comprises of 3 case-study based questions of 4 marks each.**	
36.	**Case Study – 1** The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the relation $y = 4x - \frac{1}{2}x^2$, where x is the number of days it is exposed to sunlight. Based on the above, answer the following questions :	

	(i) Find the rate of growth of the plant with respect to sunlight. *1* (ii) What is the number of days it will take for the plant to grow to the maximum height ? *2* (iii) What is the maximum height of the plant ? *1*	
Sol.	(i) $y = 4x - \frac{1}{2}x^2 \Rightarrow \frac{dy}{dx} = (4 - x)$ cm/day (ii) For maximum height, $\frac{dy}{dx} = 0 \Rightarrow x = 4$ days and $\frac{d^2y}{dx^2} < 0$ (iii) Maximum height $= y(4) = 16 - \frac{1}{2}(16) = 8$ cm	**1** **1½ +½** **1**
37.	**Case Study – 2** A cricket match is organised between two clubs P and Q for which a team from each club is chosen. Remaining players of club P and club Q are respectively sitting along the lines AB and CD, where the points are A(3, 4, 0), B(5, 3, 3), C(6, – 4, 1) and D(13, – 5, – 4). Based on the above, answer the following questions : (i) Write the direction ratios of vector $\overrightarrow{AB}$. *1* (ii) Write a unit vector in the direction of $\overrightarrow{CD}$. *1* (iii) (a) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$. *2* **OR** (iii) (b) Write a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$. *2*	

Sol.	(i) $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ $\therefore$ DRs of vector $\overrightarrow{AB}$ are $2, -1, 3$ (ii) $\overrightarrow{CD} = 7\hat{i} - \hat{j} - 5\hat{k}$, $\widehat{CD} = \frac{7}{5\sqrt{3}}\hat{i} - \frac{1}{5\sqrt{3}}\hat{j} - \frac{5}{5\sqrt{3}}\hat{k}$ (iii) (a) $\overrightarrow{AB}.\overrightarrow{CD} = 14 + 1 - 15 = 0$ $\overrightarrow{AB} \perp \overrightarrow{CD} \Longrightarrow \theta = 90°$ **OR** (iii) (b) $\overrightarrow{AB} \times \overrightarrow{CD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 7 & -1 & -5 \end{vmatrix}$ $= 8\hat{i} + 31\hat{j} + 5\hat{k}$	**1** ½ ½ **1** ½ ½ **1** **1**
38.	**Case Study – 3** A coach is training 3 players. He observes that player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and player C can hit 2 times in 3 shots. Based on the above, answer the following questions : (i) Find the probability that all three players miss the target. *1* (ii) Find the probability that all of them hit the target. *1* (iii) (a) Find the probability that only one of them hits the target. *2* **OR** (iii) (b) Find the probability that exactly two of them hit the target. *2*	
Sol.	(i) P(all will miss the target) $= \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) = \frac{1}{60}$ (ii)P(all hit the target) $= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) = \frac{2}{5}$ (iii)(a) P(only one hit the target) $= \left(\frac{4}{5}\right)\left(\frac{1}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)$ $= \frac{4}{60} + \frac{3}{60} + \frac{2}{60} = \frac{9}{60} = \frac{3}{20}$ **OR**	**1** **1** **1** ½ ½

	(iii)(b) P(Exactly two hits) $= \left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{4}\right)\left(\frac{2}{3}\right)$ $= \frac{12}{60} + \frac{6}{60} + \frac{8}{60} = \frac{26}{60} = \frac{13}{30}$	**1** ½ ½