Marking Scheme Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Supplementary Examination, 2025
SUBJECT- MATHEMATICS (041) (Q.P. CODE – 65(B)/S)

General Instructions:

1	**You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.**
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	**Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	**The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.**
5	**The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.**
6	**Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.**
7	**If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left- hand margin and encircled. This may be followed strictly.**
8	**If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.**
9	**If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**

10	**No marks to be deducted for the cumulative effect of an error. It should be penalized only once.**
11	**A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.**
12	**Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.**
13	**Ensure that you do not make the following common types of errors committed by the Examiner in the past: -** • **Leaving answer or part thereof unassessed in an answer book.** • **Giving more marks for an answer than assigned to it.** • **Wrong totaling of marks awarded on an answer.** • **Wrong transfer of marks from the inside pages of the answer book to the title page.** • **Wrong question wise totaling on the title page.** • **Wrong totaling of marks of the two columns on the title page.** • **Wrong grand total.** • **Marks in words and figures not tallying/not same.** • **Wrong transfer of marks from the answer book to online award list.** • **Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)** • **Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.**
14	**While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.**
15	**Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.**
16	**The Examiners should acquaint themselves with the guidelines given in the "Guidelines for Spot Evaluation" before starting the actual evaluation.**
17	**Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.**
18	**The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.**

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	Let A = {a, b}, then the number of reflexive relations defined on A is : (A) 16 (B) 8 (C) 4 (D) 2	
Ans	(C) 4	1
Q2.	If $A = \begin{bmatrix} x & 3 \\ 3 & x \end{bmatrix}$ and $\lvert A^3 \rvert = 343$, then x is : (A) ± 7 (B) ± 4 (C) ± 3 (D) ± 5	
Ans	(B) ± 4	1
Q3.	If A and B are square matrices both of order 3, such that $\lvert A \rvert = -3$ and $\lvert B \rvert = 2$, then $\lvert 2AB \rvert$ is equal to : (A) 48 (B) – 48 (C) – 24 (D) – 12	
Ans	(B) – 48	1
Q4.	For a skew-symmetric matrix $A = \begin{bmatrix} 0 & 7 & -4 \\ r & p & 5 \\ q & -5 & 0 \end{bmatrix}$, the value of $p + q - r$ is : (A) 11 (B) – 3 (C) – 11 (D) 3	
Ans	(A) 11	1

Q5.	If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 5 & 5 \\ 3 & 7 \end{vmatrix}$, then the value of x is : (A) 4 (B) 3 (C) 6 (D) 2	
Ans	(B) 3	1
Q6.	If A is a square matrix of order 3 such that A(adj A) = 7 I, then $\|\text{adj } A\|$ is equal to : (A) 1 (B) 343 (C) 7 (D) 49	
Ans	(D) 49	1
Q7.	If $y = \log \cos^2 \sqrt{x}$, then $\frac{dy}{dx}$ is : (A) $\frac{\tan \sqrt{x}}{\sqrt{x}}$ (B) $2 \tan \sqrt{x}$ (C) $\frac{-\tan \sqrt{x}}{\sqrt{x}}$ (D) $\frac{-\sqrt{x}}{\tan \sqrt{x}}$	
Ans	(C) $\frac{-\tan \sqrt{x}}{\sqrt{x}}$	1

Q8.	If $f(x) = \|x-1\|$, then $f'(1)$: (A) is -1 (B) is $+1$ (C) is 0 (D) does not exist	
Ans	(D) does not exist	1
Q9.	If $f(x) = x^2 + ax + 3$ is strictly increasing in the interval (3, 4), then the minimum value of a is : (A) -6 (B) -8 (C) 6 (D) 8	
Ans	(A) -6	1
Q10.	$\int \frac{1}{9x^2+6x+10}dx$ is equal to : (A) $\frac{1}{3}\tan^{-1}(3x+1)+C$ (B) $\frac{1}{9}\tan^{-1}(3x+1)+C$ (C) $\tan^{-1}\frac{3x+1}{3}+C$ (D) $\frac{1}{9}\tan^{-1}\frac{3x+1}{3}+C$	

Ans	(D) $\frac{1}{9}\tan^{-1}\frac{3x+1}{3}+C$	1
Q11.	The value of $\int_{-2}^{2}\sin^5 x\cos x\ dx$ is : (A) $\frac{64}{3}$ (B) 0 (C) $2\sin^6 2$ (D) $\sin^6(-2)-\sin^6 2$	
Ans	(B) 0	1
Q12.	The area enclosed by the curve $y=\sqrt{4-x^2}$ and the coordinate axes in the first quadrant is : (A) 2π sq. units (B) π sq. units (C) $\frac{\pi}{2}$ sq. units (D) 4π sq. units	
Ans	(B) π sq. units	1
Q13.	The general solution of the differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is : (A) $\tan^{-1}(y-x)=C$ (B) $\tan^{-1}y+\tan^{-1}x=C$ (C) $\tan^{-1}\frac{y}{2}-\tan^{-1}\frac{x}{2}=C$ (D) $\tan^{-1}y=\tan^{-1}x+C$	
Ans	(D) $\tan^{-1}y=\tan^{-1}x+C$	1

Q14.	The integrating factor for solving the differential equation $\tan x \frac{dy}{dx} + y = x^2, (x \neq 0)$ is : (A) e^x (B) $\tan x$ (C) $\sin x$ (D) $\frac{1}{\sin x}$	
Ans	(C) $\sin x$	1
Q15.	The value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector, is : (A) 1 (B) $\pm \sqrt{3}$ (C) $\pm \frac{1}{\sqrt{3}}$ (D) ± 3	
Ans	(C) $\pm \frac{1}{\sqrt{3}}$	1
Q16.	The solution set of the inequality $5x + 4y < 7$ is : (A) Open half-plane containing the origin. (B) Whole xy-plane except the points lying on the line $5x + 4y = 7$. (C) Open half-plane not containing the origin. (D) Closed half-plane not containing the origin.	
Ans	(A) Open half-plane containing the origin.	1
Q17.	Of all the points of the feasible region of an LPP, for maximum or minimum values of objective function, the points lie : (A) inside the feasible region (B) at the boundary line of the feasible region (C) at the corners of the feasible region (D) at the points of intersection of the feasible region with x-axis	
Ans	(C) at the corners of the feasible region	1

Q18.	If P(A) = 0·3, P(B) = 0·4 and P(A ∪ B) = 0·6, then P(A | B) + P(B | A) equals : (A) $\frac{5}{12}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{7}{12}$	
Ans	(D) $\frac{7}{12}$	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Q19.	*Assertion (A) :* If A is a skew-symmetric matrix of order 3, then $\|A\| = 0$. *Reason (R) :* If A is a square matrix of order 3, then $\|A\| = \|A'\|$.	
Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1
Q20.	*Assertion (A) :* $\langle \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}} \rangle$ cannot be the direction cosines of a line. *Reason (R) :* If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.	
Ans	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1

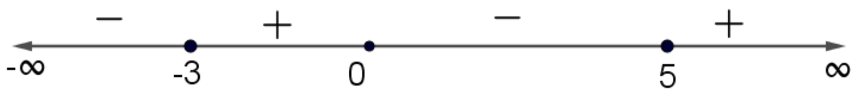
SECTION B This section comprises very short answer (VSA) type questions of **2 marks each**.		
Q21.	Evaluate : $\sec^2(\tan^{-1} 3) + \text{cosec}^2(\cot^{-1} 2)$	
Ans	$\sec^2(\tan^{-1} 3) + \text{cosec}^2(\cot^{-1} 2) = 1 + \tan^2(\tan^{-1} 3) + 1 + \cot^2(\cot^{-1} 2)$ $= 1 + 3^2 + 1 + 2^2 = 15$	1 1
Q22.	(a) Find the value of k, so that f(x) = $\begin{cases} \dfrac{k\cos x}{\pi - 2x}, & \text{if } x \neq \dfrac{\pi}{2} \\ 3, & \text{if } x = \dfrac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$. **OR** (b) Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$.	
Ans(a)	$f(x)$ is continuous at $x = \frac{\pi}{2}$ $\Rightarrow \lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^+} f(x) = f\left(\frac{\pi}{2}\right)$ $\Rightarrow \lim_{h\to 0} \frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi - 2\left(\frac{\pi}{2}-h\right)} = 3$ $\Rightarrow \frac{k}{2}\lim_{h\to 0}\frac{\sin h}{h} = 3$ $\Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$	1 ½ ½
OR		
Ans (b)	$y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$ $= \frac{\pi}{4} + \frac{x}{2}$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}$	1 ½ ½
Q23.	The radius of a cylinder is increasing at the rate of 3 cm/s, and its height is decreasing at the rate of 5 cm/s. Find the rate of change of its volume, when radius is 4 cm and height is 7 cm.	

Ans	$\frac{dr}{dt} = 3$ cm/s and $\frac{dh}{dt} = -5$ cm/s $V = \pi r^2 h$ $\Rightarrow \frac{dv}{dt} = \pi\left(2r\frac{dr}{dt}\right)h + \pi r^2\frac{dh}{dt}$ $\Rightarrow \left.\frac{dV}{dt}\right]_{r=4,h=7} = 4\pi\big(2\times3\times7 + 4\times(-5)\big) = 4\pi\times22 = 88\pi\ \text{cm}^3/\text{s}$	½ 1 ½
Q24.	(a) If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $\lvert\vec{a}\rvert = 2$, $\lvert\vec{b}\rvert = 3$ and $\lvert\vec{c}\rvert = 4$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$. **OR** (b) Let $\vec{a} = 2\hat{i}+3\hat{j}+4\hat{k}$, $\vec{b} = 4\hat{i}-3\hat{j}+9\hat{k}$ and $\vec{c} = 3\hat{i}-2\hat{j}+6\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d} = 39$.	
Ans(a)	As $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ $\Rightarrow \left(\vec{a}+\vec{b}+\vec{c}\right)^2 = 0$ $\Rightarrow \vec{a}^2+\vec{b}^2+\vec{c}^2+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$ $\Rightarrow 4+9+16+2(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}) = 0$ $\Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = -\frac{29}{2}$	 1 ½ ½
	OR	
Ans(b)	Vector $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$ So $\vec{d} = \lambda(\vec{a}\times\vec{b})$ $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 3 & 4\\ 4 & -3 & 9\end{vmatrix} = 39\hat{\imath}-2\hat{\jmath}-18\hat{k}$ $\Rightarrow \vec{d} = \lambda(39\hat{\imath}-2\hat{\jmath}-18\hat{k})$ As $\vec{c}.\vec{d} = 39$ $\Rightarrow \big(3\times39\lambda - 2\times(-2\lambda) + 6\times(-18\lambda)\big) = 39$ $\Rightarrow 13\lambda = 39 \Rightarrow \lambda = 3$ $\Rightarrow \vec{d} = 3(39\hat{\imath}-2\hat{\jmath}-18\hat{k}) = 117\hat{\imath}-6\hat{\jmath}-54\hat{k}$	 ½ ½ ½ ½

Q25.	If the equations of a line are $\frac{x-2}{2} = \frac{2y-5}{-3}$, $z = -1$, then find the direction ratios of the line and also a point on the line.	
Ans	$\frac{x-2}{2} = \frac{2y-5}{-3}, z = -1 \Rightarrow \frac{x-2}{2} = \frac{y-\frac{5}{2}}{-\frac{3}{2}} = \frac{z+1}{0}$ Direction Ratios $\left(2, -\frac{3}{2}, 0\right)$ or $(4, -3, 0)$ A Point on the line = $\left(2, \frac{5}{2}, -1\right)$ or any other point on the given line	1 ½ ½

SECTION C

This section comprises short answer (SA) type questions of **3 marks each**.

Q26.	Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is : (a) strictly increasing. (b) strictly decreasing.	
Ans	$f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51 \Rightarrow f'(x) = 6x^3 - 12x^2 - 90x$ For critical points $f'(x) = 0$ $\Rightarrow 6x^3 - 12x^2 - 90x = 0$ $\Rightarrow 6x(x^2 - 2x - 15) = 0$ $\Rightarrow 6x(x-5)(x+3) = 0 \Rightarrow x = -3, 0, 5$  $f(x)$ is strictly increasing in $(-3,0)$ *and* $(5,\infty)$ ($f(x)$ is strictly decreasing in $(-\infty,-3)$ *and* $(0,5)$ Note: Closed intervals are also acceptable.	1 1 1
Q27.	(a) Find : $\int \sin^3 x \cdot \cos^4 x \, dx$ **OR** (b) Evaluate : $\int_{-2}^{1} \lvert x^3 - x \rvert \, dx$	

Ans(a)	$\int \sin^3 x \cos^4 x\, dx = \int (1-\cos^2 x)\cos^4 x \sin x\, dx$ $= -\int (1-t^2)t^4\, dt$ (putting $\cos x = t$) $= -\int (t^4 - t^6)\, dt$ $= \int (t^6 - t^4)\, dt$ $= \frac{t^7}{7} - \frac{t^5}{5} + C$ $= \frac{1}{7}\cos^7 x - \frac{1}{5}\cos^5 x + C$	½ 1 1 ½
	OR	
Ans(b)	$\lvert x^3 - x\rvert = \lvert x(x-1)(x+1)\rvert$ $= \begin{cases} -(x^3-x), & if\ -2 < x < -1 \\ x^3 - x, & if\ -1 < x < 0 \\ -(x^3-x), & if\ 0 < x < 1 \end{cases}$ $\int_{-2}^{1}\lvert x^3 - x\rvert\, dx = \int_{-2}^{-1}(x - x^3)\, dx + \int_{-1}^{0}(x^3 - x)\, dx + \int_{0}^{1}(x - x^3)\, dx$ $= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-2}^{-1} + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{0}^{1}$ $= \left[\left(\frac{1}{2} - \frac{1}{4}\right) - (2-4)\right] + \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] + \left[\left(\frac{1}{2} - \frac{1}{4}\right) - 0\right]$ $= \frac{9}{4} + \frac{1}{4} + \frac{1}{4} = \frac{11}{4}$	 1 1 1
Q28.	(a) Solve the differential equation $y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$. **OR** (b) Find the particular solution of the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.	
Ans(a)	$y + x\frac{dy}{dx} = x - y\frac{dy}{dx} \Rightarrow \frac{dy}{dx}(x+y) = x - y \Rightarrow \frac{dy}{dx} = \frac{x-y}{x+y}$ ….(i) Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ Then eqn(i) reduces to $v + x\frac{dv}{dx} = \frac{1-v}{1+v} \Rightarrow x\frac{dv}{dx} = \frac{1-v}{1+v} - v = \frac{1-v-v-v^2}{1+v} = \frac{-(v^2+2v-1)}{v+1}$ $\Rightarrow \frac{1}{2}\int \frac{2v+2}{v^2+2v-1}\, dv = -\int \frac{dx}{x} \Rightarrow \frac{1}{2}\log\lvert v^2 + 2v - 1\rvert = -\log\lvert x\rvert + \log C$ $\Rightarrow \left(\left(\frac{y}{x}\right)^2 + 2\left(\frac{y}{x}\right) - 1\right)^{\frac{1}{2}} = \frac{C}{x} \Rightarrow y^2 + 2xy - x^2 = C^2 = K$ (a constant)	½ 1 1 ½
	OR	
Ans(b)	$x\frac{dy}{dx} + y = x\cos x + \sin x \Rightarrow \frac{dy}{dx} + \frac{y}{x} = \cos x + \frac{1}{x}\sin x$ $I.F. = e^{\int \frac{1}{x}dx} = e^{\log x} = x$	½ ½

	Solution is $y.x = \int \left(\cos x + \frac{1}{x}\sin x\right) x\, dx = \int (x\cos x + \sin x)\ dx$ $\Rightarrow yx = x(\sin x) - \int 1.\sin x\ dx + \int \sin x\ dx = x\sin x + C$ Now $y = 1$ when $x = \frac{\pi}{2}$ $1.\frac{\pi}{2} = \frac{\pi}{2}\sin\frac{\pi}{2} + C \Rightarrow 1 = 1 + C \Rightarrow C = 0$ Thus, particular solution is $yx = x\sin x \Rightarrow y = \sin x$	½ 1 ½
Q29.	Using vectors, find the area of triangle ABC with vertices A(4, 3, 3), B(5, 5, 6) and C(4,7, 6).	
Ans	$\overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k},\ \overrightarrow{AC} = 4\hat{\jmath} + 3\hat{k},$ $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix} = -6\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ So, the area of the triangle $= \frac{1}{2}\left\lvert \overrightarrow{AB} \times \overrightarrow{AC} \right\rvert$ $= \frac{1}{2}\left\lvert -6\hat{\imath} - 3\hat{\jmath} + 4\hat{k} \right\rvert = \frac{1}{2}\sqrt{36 + 9 + 16} = \frac{1}{2}\sqrt{61}$	1 1 1
Q30.	The corner points of the feasible region determined by the system of linear constraints for an LPP are (0, 10), (5, 5), (15, 15) and (0, 20). Let z = ax + by, where a, b > 0 be the objective function. Find the condition on a and b so that maximum of z occurs at both the points (15, 15) and (0, 20).	
Ans	$Z = ax + by$ $Z_{(15,15)} = Z_{(0,20)}$ $\Rightarrow 15a + 15b = 20b$ $\Rightarrow 15a = 5b$ $\Rightarrow 3a = b$	2 1

Q31.	(a) Two balls are drawn at random without replacement from a box containing 3 black and 7 red balls. Find the probability that : (i) both balls are red. (ii) first ball is black and the second is red. **OR** (b) Two cards are drawn successively with replacement of a well-shuffled pack of 52 playing cards. Find the probability distribution of the number of face cards.	
Ans(a)	(i) $P(both\ balls\ are\ red) = \frac{7}{10} \times \frac{6}{9} = \frac{7}{15}$ (ii) $P(first\ ball\ is\ black\ and\ second\ is\ red) = \frac{3}{10} \times \frac{7}{9} = \frac{7}{30}$	1+ ½ 1+ ½
	OR	
Ans(b)	Let the random variable X: Number of Face Cards Then X = 0, 1, 2 Total Cards = 52, total face cards = 12 $P(a\ face\ card) = \frac{12}{52} = \frac{3}{13},\ \ P(not\ a\ face\ card) = 1 - \frac{3}{13} = \frac{10}{13}$	½ ½ + ½

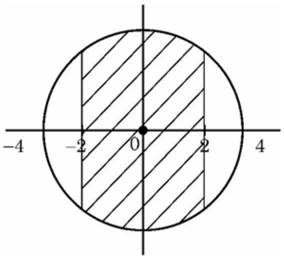
X	P(X)	
0	$\frac{10}{13} \times \frac{10}{13} = \frac{100}{169}$	½
1	$2 \times \frac{10}{13} \times \frac{3}{13} = \frac{60}{169}$	½
2	$\frac{3}{13} \times \frac{3}{13} = \frac{9}{169}$	½

SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

Q32.	Let A be the set of all positive integers and a relation R on A × A is defined by (a, b) R (c, d) ⇔ ad = bc, for all (a, b), (c, d) ∈ A × A. Show that R is an equivalence relation on A × A.	
Ans	$A = Z^+, R\ on\ A \times A\ is\ defined\ as\ (a,b)R(c,d) \Leftrightarrow ad = bc$	

	Reflexive: Let for $a, b \in Z^+$; $(a,b) \in A \times A$ Then $ab = ba$ $\Rightarrow (a,b)R(a,b)\ \forall (a,b) \in A \times A$ So, R is reflexive. **Symmetric:** Let $(a,b)\ R\ (c,d)$, $where\ (a,b),(c,d) \in A \times A$ $\Rightarrow ad = bc \Rightarrow cb = da \Rightarrow (c,d)R(a,b)$ So, R is symmetric. **Transitive:** Now, $let\ (a,b)R\ (c,d)\ and\ (c,d)R\ (e,f)\ ,where\ (a,b),(c,d),(e,f) \in A \times A$ $\Rightarrow ad = bc\ and\ cf = de \Rightarrow ad.cf = bc.de \Rightarrow af = be$ $\Rightarrow (a,b)\ R\ (e,f)$ So, R is transitive. Hence R is an equivalence relation.	1 ½ 1 ½ 1 ½ ½
Q33.	(a) If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$, $x^2 \le 1$, then find $\frac{dy}{dx}$. **OR** (b) If $y = \left(x+\sqrt{1+x^2}\right)^n$, then show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2y.$	
Ans(a)	$y = tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right]$ Let $x^2 = cos\ \theta$ Then $y = \tan^{-1}\left[\frac{\sqrt{2}\cos\left(\frac{\theta}{2}\right)+\sqrt{2}\sin\left(\frac{\theta}{2}\right)}{\sqrt{2}\cos\left(\frac{\theta}{2}\right)-\sqrt{2}\sin\left(\frac{\theta}{2}\right)}\right]$ $= \tan^{-1}\left[\frac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)}\right]$ $= \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right] = \frac{\pi}{4}+\frac{\theta}{2}$ $= \frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$ $\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(-\frac{1}{\sqrt{1-x^4}}\right)2x = -\frac{x}{\sqrt{1-x^4}}$	½ 1 2 ½ 1
	OR	
Ans(b)	$y = \left(x+\sqrt{1+x^2}\right)^n$ $\Rightarrow \frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left(1+\frac{2x}{2\sqrt{1+x^2}}\right)$ $\Rightarrow \frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^{n-1}\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}\right)$	 2

	$\Rightarrow \sqrt{1+x^2}\frac{dy}{dx} = n\left(x+\sqrt{1+x^2}\right)^n = ny$ Again, differentiating both sides w.r.t. x, we get $\sqrt{1+x^2}\frac{d^2y}{dx^2} + \frac{2x}{2\sqrt{1+x^2}}\frac{dy}{dx} = n\frac{dy}{dx}$ $\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n\sqrt{1+x^2}\frac{dy}{dx} = n.ny = n^2y$ Hence Proved.	1 1 1
Q34.	Using integration, find the area of the region bounded between the lines x = – 2, x = 2 and the circle $x^2 + y^2 = 16$.	
Ans	Required Area $= 4\int_0^2 \sqrt{16 - x^2}\,dx$ $= 4\left[\frac{x}{2}\sqrt{4^2 - x^2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^2$ $= 4\left[\left(\sqrt{12} - 0\right) + 8\left(\sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1} 0\right)\right]$ $= 4\left(2\sqrt{3} + 8 \times \frac{\pi}{6}\right)$ $= \left(8\sqrt{3} + \frac{16\pi}{3}\right)$ 	**2** **2** **1**
Q35.	(a) Write the nature of the lines $\frac{x-1}{4} = \frac{y-2}{6} = \frac{z-3}{8}$ and $\frac{x-2}{2} = \frac{y-4}{3} = \frac{z-5}{4}$. Also, find the shortest distance between them. **OR** (b) Show that the lines $\vec{r} = (2\hat{j} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k})$ intersect. Also find their point of intersection.	
Ans(a)	Given lines are $l_1: \frac{x-1}{4} = \frac{y-2}{6} = \frac{z-3}{8}$ *and* $l_2: \frac{x-2}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ DRs of two lines are 4, 6, 8 and 2, 3, 4 Here $\frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, So, lines are parallel Given lines in vector form are $l_1: \vec{r} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$ *and* $l_2: \vec{r} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k})$ Here $\vec{a_1} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{a_2} = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}, \vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$ $\vec{a_2} - \vec{a_1} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$	**1** **1** **1**

$(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 2 & 2\\ 2 & 3 & 4\end{vmatrix}=2\hat{\imath}-\hat{k}$ — **1 ½**

So, shortest distance = $\frac{|(\overrightarrow{a_2}-\overrightarrow{a_1})\times\vec{b}|}{|\vec{b}|}=\frac{\sqrt{5}}{\sqrt{29}}$ — **½**

OR

Ans(b) Given lines are

$l_1: \vec{r}=2\hat{\jmath}-3\hat{k}+\lambda(\hat{\imath}+2\hat{\jmath}+3\hat{k})$ and $l_2: \vec{r}=2\hat{\imath}+6\hat{\jmath}+3\hat{k}+\mu(2\hat{\imath}+3\hat{\jmath}+4\hat{k})$

Any point on l_1 is $(\lambda, 2+2\lambda, -3+3\lambda)$ — 1

Any point on l_2 is $(2+2\mu, 6+3\mu, 3+4\mu)$ — 1

If lines intersect then for some value of λ and μ

$(\lambda, 2+2\lambda, -3+3\lambda)=(2+2\mu, 6+3\mu, 3+4\mu)$

$\Rightarrow \lambda=2+2\mu, 2+2\lambda=6+3\mu, -3+3\lambda=3+4\mu$ — 1

$\Rightarrow \lambda-2\mu=2, 2\lambda-3\mu=4, 3\lambda-4\mu=6$

Solving first two equations $\lambda=2$ and $\mu=0$ — 1

And these values satisfy the third equation

So, lines are intersecting and point of intersection is (2,6,3) — 1

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.

Case Study – 1

To promote the making of toilets in villages, an NGO hired an agency for generating awareness for the cause through house calls, letters and announcements through speakers.

The cost per mode of communication is given below :

Cost per visit / communication	House calls	Letters	Announcements
₹	15	10	25

The number of contacts made in two villages X and Y were as follows :

Village	Houses visited	Letters sent	Number of announcements
X	100	150	110
Y	150	200	150

	Using the above information, answer the following questions : (i) Write A, the matrix for cost per visit/communication. *1* (ii) Write B, the matrix representing number of contacts in two villages, X and Y. *1* (iii) (a) Find the cost (in ₹) incurred by the NGO for village X. *2* **OR** (iii) (b) Find the cost (in ₹) incurred by the NGO for village Y. *2*	
Ans	(i) $A = (15 \quad 10 \quad 25)$ or $A = \begin{pmatrix} 15 \\ 10 \\ 25 \end{pmatrix}$	1
	(ii) $B = \begin{pmatrix} 100 & 150 \\ 150 & 200 \\ 110 & 150 \end{pmatrix}$ OR $B = \begin{pmatrix} 100 & 150 & 110 \\ 150 & 200 & 150 \end{pmatrix}$	1
	(iii)(a) $AB = (15 \quad 10 \quad 25)\begin{pmatrix} 100 & 150 \\ 150 & 200 \\ 110 & 150 \end{pmatrix} = (5750 \quad 8000)$ or $BA = \begin{pmatrix} 100 & 150 & 110 \\ 150 & 200 & 150 \end{pmatrix}\begin{pmatrix} 15 \\ 10 \\ 25 \end{pmatrix} = \begin{pmatrix} 5750 \\ 8000 \end{pmatrix}$ Cost incurred by the NGO for village X = ₹ 5750	1 + ½ ½
	OR (iii)(b) $AB = (15 \quad 10 \quad 25)\begin{pmatrix} 100 & 150 \\ 150 & 200 \\ 110 & 150 \end{pmatrix} = (5750 \quad 8000)$ or $BA = \begin{pmatrix} 100 & 150 & 110 \\ 150 & 200 & 150 \end{pmatrix}\begin{pmatrix} 15 \\ 10 \\ 25 \end{pmatrix} = \begin{pmatrix} 5750 \\ 8000 \end{pmatrix}$ Cost incurred by the NGO for village Y = ₹ 8000	1 + ½ ½

Q37.	**Case Study – 2** A magazine company circulates its magazine on a monthly basis in a city. It has 10,000 readers on its list and collects fixed charges of ₹ 4,000 per reader annually. The company proposes to increase the annual subscription, but on the basis of a survey result, it predicted that for every increase of ₹ 5, ten readers will discontinue the service of this magazine company. Based on the above information, answer the following questions : (i) Let the company increase ₹ x, then find the function R(x) representing the earnings of the company. *1* (ii) Find $\frac{d}{dx}(R(x))$. *1* (iii) (a) What subscription increase will bring maximum earnings for the company ? *2* **OR** (iii) (b) What will be the maximum value of R(x) ? *2*	
Ans	(i) $R(x) = (4000 + x)(10000 - 2x)$	1
	(ii) $\frac{d}{dx}\left(R(x)\right) = (4000 + x)(-2) + (10000 - 2x).1$ $= -8000 - 2x + 10000 - 2x$ $= 2000 - 4x$	1
	(iii)(a) $\frac{d}{dx}\left(R(x)\right) = 0 \Rightarrow 2000 - 4x = 0 \Rightarrow x = 500$ Now $\frac{d^2(R(x))}{dx^2} = -4 < 0$ So, R is maximum at $x = 500$	1+ ½ ½
	OR	
	(iii)(b) $\frac{d}{dx}\left(R(x)\right) = 0 \Rightarrow 2000 - 4x = 0 \Rightarrow x = 500$ $\frac{d^2(R(x))}{dx^2} = -4 < 0$ So, R is maximum at $x = 500$ Maximum Value of $R(x) = (4000 + 500)(10000 - 1000)$ $= 4500 \times 9000$ $=$ ₹ 4,05,00,000	1 ½ ½

Q38.	**Case Study – 3** A coach is training 3 players. He observes that player A can hit the target 4 times in 5 shots, player B can hit the target 3 times in 4 shots and player C can hit the target 2 times in 3 shots. Based on the above information, answer the following questions : If they all try independently, find the probability that : (i) exactly two of them hit the target. *2* (ii) at least one of them hits the target. *2*	
Ans	Let the events be A: Player A can hit the target B: Player B can hit the target C: Player C can hit the target Then $P(A) = \frac{4}{5}, P(B) = \frac{3}{4}, P(C) = \frac{2}{3}$ and $P(A') = \frac{1}{5}, P(B') = \frac{1}{4}, P(C') = \frac{1}{3}$ (i) Probability that exactly two of them hit the target $= P(A)P(B)P(C') + P(A)P(B')P(C) + P(A')P(B)P(C)$ $= \frac{4}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{4}{5} \times \frac{1}{4} \times \frac{2}{3} + \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}$ $= \frac{1}{5} + \frac{2}{15} + \frac{1}{10} = \frac{13}{30}$ (ii) Probability that atleast one of them hits the target $= 1 -$ Probability that none of them hits the target $= 1 - \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}$ $= 1 - \frac{1}{60} = \frac{59}{60}$	 1 ½ ½ 1 1