Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
MATHEMATICS PAPER CODE 65(B)
(FOR VISUALLY IMPAIRED CANDIDATES ONLY)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives the impression that answer is correct, and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".**
10	**In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

14	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

Senior Secondary School Examination, 2022-23

MATHEMATICS (Subject Code–041)

[Paper Code : 65(B)]

Maximum Marks : 80

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each**.	
Q1.	The domain of the function $\cos^{-1} x$ is : (a) $[0, \pi]$ (b) $(-1, 1)$ (c) $[-1, 1]$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	
A1.	(c) $[-1, 1]$	1
Q2.	In a 3×3 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{1}{2}\lvert -3i + j \rvert$, the element a_{31} is : (a) -4 (b) 5 (c) 4 (d) 8	
A2.	(c) 4	1
Q3.	If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then $2x - y$ is equal to : (a) 3 (b) 13 (c) -3 (d) 0	
A3.	(a) 3	1

Q4.	If $A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$, then A^{-1} is given by : (a) $\begin{bmatrix} 5 & -2 \\ -3 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$	
A4.	(b) $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$	1
Q5.	In the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, M_{23} is : (where M_{ij} denotes the minor of element a_{ij}) (a) 7 (b) -13 (c) 13 (d) -7	
A5.	(c) 13	1
Q6.	If $y = \sec(\tan^{-1} x)$, then $\frac{dy}{dx}$ at $x = 1$ is equal to : (a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 1 (d) $\frac{1}{2}$	
A6.	(b) $\frac{1}{\sqrt{2}}$	1

Q7.	The value of k for which the function f given by $f(x)=\begin{cases}\dfrac{k\cos x}{\pi-2x}, & \text{if } x\neq\dfrac{\pi}{2}\\ 5, & \text{if } x=\dfrac{\pi}{2}\end{cases}$ is continuous at $x=\dfrac{\pi}{2}$, is : (a) 6 (b) 5 (c) $\dfrac{5}{2}$ (d) 10	
A7.	(d) 10	1
Q8.	$\int \dfrac{dx}{\sin^2 2x \cdot \cos^2 2x}$ equals : (a) $\dfrac{1}{2}[\tan 2x + \cot 2x] + C$ (b) $\tan 2x - \cot 2x + C$ (c) $\dfrac{1}{2}[\tan 2x - \cot 2x] + C$ (d) $\dfrac{1}{2}[\cot 2x - \tan 2x] + C$	
A8.	(c) $\dfrac{1}{2}[\tan 2x - \cot 2x] + C$	1
Q9.	$\int_{-\pi/4}^{\pi/4} \sin^3 x\, dx$ equals : (a) $2\int_0^{\pi/4} \sin^3 x\, dx$ (b) 0 (c) 1 (d) $\int_0^{\pi/4} \sin^3 x\, dx$	
A9.	(b) 0	1

Q10.	The general solution of the differential equation $\frac{dy}{dx} = e^{x-y}$ is : (a) $e^x + e^y = C$ (b) $e^x - e^{-y} = C$ (c) $-e^x - e^y = C$ (d) $e^x - e^y = C$	
A10.	(d) $e^x - e^y = C$	1
Q11.	The sum of the order and the degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + 3x\left(\frac{d^2y}{dx^2}\right)^4 = \log x$, is : (a) 5 (b) 6 (c) 7 (d) 4	
Q11.	(a) 5	1
Q12.	Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. $\vec{a} + \vec{b}$ is a unit vector, if : (a) $\theta = \frac{\pi}{3}$ (b) $\theta = \frac{\pi}{4}$ (c) $\theta = \frac{\pi}{2}$ (d) $\theta = \frac{2\pi}{3}$	
A12.	(d) $\theta = \frac{2\pi}{3}$	1
Q13.	If $(2\hat{i} + 6\hat{j} - 22\hat{k}) \times (\hat{i} + \lambda\hat{j} + \mu\hat{k}) = \vec{0}$, then $\lambda - \mu$ is equal to : (a) -8 (b) -14 (c) 14 (d) 8	
A13.	(c) 14	1

Q14.	If the position vectors of two points A and B are $\hat{i} + 2\hat{j} - 3\hat{k}$ and $-\hat{i} - 2\hat{j} + \hat{k}$ respectively, then the direction cosines of the vector $\overrightarrow{BA}$ are : (a) $\frac{2}{6}, -\frac{4}{6}, -\frac{4}{6}$ (b) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$ (c) $\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$ (d) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$	
A14.	(b) $\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}$	1
Q15.	The value of λ for which the lines $\frac{x-5}{7} = \frac{2-y}{5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{2y-1}{\lambda} = \frac{z}{3}$ are at right angles, is : (a) 2 (b) 4 (c) -4 (d) -2	
A15.	(b) 4	1
Q16.	The solution set of the inequation $2x + y \geq 5$ is : (a) half plane that contains the origin (b) open half plane not containing the origin and not containing the points on the line $2x + y = 5$. (c) whole xy-plane except the points lying on the line $2x + y = 5$. (d) open half plane not containing the origin, but containing the points on the line $2x + y = 5$.	
A16.	(d) open half plane not containing the origin, but containing the points on the line $2x + y = 5$.	1
Q17.	The minimum value of $z = 3x + 8y$ subject to the constraints $x \leq 20$, $y \geq 10$ and $x \geq 0$, $y \geq 0$ is : (a) 80 (b) 140 (c) 0 (d) 60	
A17.	(a) 80	1

Q18.	Two events A and B will be independent, if : (a) A and B are mutually exclusive (b) $P(A) = P(B)$ (c) $P(A'B') = [1 - P(A)]\ [1 - P(B)]$ (d) $P(A) + P(B) = 1$	
A18.	(c) $P(A'B') = [1 - P(A)]\ [1 - P(B)]$	1

*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

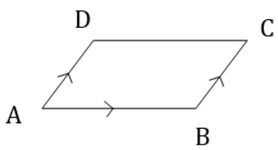
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

Q19.	*Assertion (A) :* Matrix $A = \begin{bmatrix} 0 & -3 & 5 \\ 3 & 0 & -2 \\ -5 & 2 & 0 \end{bmatrix}$ is a skew-symmetric matrix. *Reason (R) :* If $A' = -A$, then A is a skew-symmetric matrix.	
A19.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).	1
Q20.	*Assertion (A) :* The vector equation of a line passing through the points A(–1, 0, 2) and B(3, 4, 6) is $\vec{r} = -\hat{i} + 2\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$. *Reason (R) :* The equation of a line passing through a point with position vector $\vec{a}$ and parallel to a vector $\vec{b}$, is $\vec{r} = \vec{a} + \lambda \vec{b}$.	
A20.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). **OR** (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).	1

SECTION B This section comprises very short answer (VSA) type questions of **2 marks each**.		
Q21.	(a) Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. **OR** (b) Check the injectivity and surjectivity of the function $f : N \to N$, given by $f(x) = x^3$.	
A21.	$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ $= \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right) = -\frac{\pi}{3}$	1+1
	OR	
	For Injectivity Let $x_1, x_2 \in N$; Let $f(x_1) = f(x_2)$ $\Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2 \Rightarrow f$ is injective function.	1
	For Surjectivity Let $y \in N$, then $f(x) = y \Rightarrow x^3 = y \Rightarrow x = y^{1/3} \notin N$ for some y $\Rightarrow f$ is not surjective.	1
	Note: Student can justify f as non-surjective by taking some counter examples also.	
Q22.	If $\cos y = x \cos(a + y)$, and $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.	
A22.	$x = \frac{\cos y}{\cos(a+y)}$	½
	$\Rightarrow \frac{dx}{dy} = \frac{\cos(a+y).(-\sin y) - \cos y.[-\sin(a+y)]}{\cos^2(a+y)}$	1
	$\Rightarrow \frac{dx}{dy} = \frac{\sin(a+y-y)}{\cos^2(a+y)}$	
	$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$	½
Q23.	Find the projection of the vector $7\hat{i} - \hat{j} + 8\hat{k}$ on the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.	

A23.	$p=\frac{(7\hat{i}-\hat{j}+8\hat{k}).(\hat{i}+2\hat{j}+2\hat{k})}{\lvert\hat{i}+2\hat{j}+2\hat{k}\rvert}$ $=\frac{(7)(1)+(-1)(2)+(8)(2)}{\sqrt{1^2+2^2+2^2}}=7$	1 1
Q24.	(a) In a parallelogram ABCD, the sides AB and AD are represented by the vectors $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$ respectively. Find the unit vector parallel to its diagonal $\vec{AC}$. **OR** (b) Find the angle between the pair of lines given by $\vec{r}=\hat{i}+2\hat{j}-2\hat{k}+\lambda(\hat{i}-2\hat{j}-2\hat{k})$ and $\vec{r}=3\hat{i}-5\hat{j}+\hat{k}+\mu(3\hat{i}+2\hat{j}-6\hat{k})$.	
A24.	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AB}+\overrightarrow{AD}=3\hat{i}-6\hat{j}+2\hat{k}$ Required unit vector $=\frac{1}{7}(3\hat{i}-6\hat{j}+2\hat{k})$  **OR** $\cos\theta=\frac{(\hat{i}-2\hat{j}-2\hat{k}).(3\hat{i}+2\hat{j}-6\hat{k})}{\lvert\hat{i}-2\hat{j}-2\hat{k}\rvert.\lvert 3\hat{i}+2\hat{j}-6\hat{k}\rvert}$ $=\frac{(1)(3)+(-2)(2)+(-2)(-6)}{\sqrt{(1)^2+(-2)^2+(-2)^2}\sqrt{(3)^2+(2)^2+(-6)^2}}=\frac{11}{21}$ $\Rightarrow\theta=\cos^{-1}\left(\frac{11}{21}\right)$	1 1 1 ½ ½
Q25.	The radius of an air bubble is increasing at the rate of 0·5 cm/s. At what rate is the surface area of the bubble increasing when the radius is 1·5 cm ?	
A25.	$\frac{dr}{dt}=0.5\text{cm/s}$ (given) Now, $S=4\pi r^2$ $\Rightarrow\frac{dS}{dt}=8\pi r.\frac{dr}{dt}$ $\therefore\left.\frac{dS}{dt}\right]_{r=1.5\text{ cm}}=8\pi(1.5)(0.5)=6\pi\text{ cm}^2/\text{s}$	½ 1 ½

SECTION C		
This section comprises short answer (SA) type questions of **3 marks each**.		
Q26.	Find : $$\int \frac{dx}{1+\cot x}$$	
A26.	$I=\int \frac{dx}{1+\cot x}=\int \frac{\sin x}{\sin x+\cos x}dx$ $=\frac{1}{2}\int \frac{(\sin x+\cos x)-(\cos x-\sin x)}{\sin x+\cos x}dx$ $=\frac{1}{2}\int 1.dx+\frac{1}{2}\int \frac{\sin x-\cos x}{\sin x+\cos x}dx$ $=\frac{x}{2}-\frac{1}{2}\log\lvert\sin x+\cos x\rvert+C$	 1 ½ $\frac{1}{2}+1$
Q27.	(a) Evaluate : $$\int_{\pi/2}^{\pi} e^{x}\left(\frac{1-\sin x}{1-\cos x}\right)dx$$ **OR** (b) Evaluate : $$\int_{0}^{\pi/4} \log(1+\tan x)\,dx$$	
A27.	$I=\int_{\frac{\pi}{2}}^{\pi} e^{x}\left(\frac{1-\sin x}{1-\cos x}\right)dx$ $=\int_{\frac{\pi}{2}}^{\pi} e^{x}\left(\frac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^{2}\frac{x}{2}}\right)dx$ $=\int_{\frac{\pi}{2}}^{\pi} e^{x}\left(\frac{1}{2}\operatorname{cosec}^{2}\frac{x}{2}-\cot\frac{x}{2}\right)dx$ $=-\left[e^{x}\cot\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi}$ $=e^{\frac{\pi}{2}}$ **OR**	 1 ½ 1 ½

	$I=\int_0^{\frac{\pi}{4}}\log(1+\tan x)\,dx$	
	$=\int_0^{\frac{\pi}{4}}\log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$	1
	$=\int_0^{\frac{\pi}{4}}\log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx=\int_0^{\frac{\pi}{4}}\log\left(\frac{2}{1+\tan x}\right)dx$	1
	$=\int_0^{\frac{\pi}{4}}\log(2)\,dx-\int_0^{\frac{\pi}{4}}\log(1+\tan x)\,dx$	
	$\Rightarrow I=\log 2[x]_0^{\pi/4}-I$	½
	$\Rightarrow I=\frac{\pi}{8}\log 2$	½
Q28.	Find : $$\int\frac{x}{(x^2+1)(x-1)}\,dx$$	
A28.	$I=\int\frac{x}{(x^2+1)(x-1)}dx$	
	Let $\frac{x}{(x^2+1)(x-1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$	1
	$\Rightarrow x=A(x^2+1)+(Bx+C)(x-1)$	
	$\Rightarrow A=\frac{1}{2},B=-\frac{1}{2},C=\frac{1}{2}$	
	$I=\frac{1}{2}\int\frac{dx}{x-1}-\frac{1}{4}\int\frac{2x}{x^2+1}dx+\frac{1}{2}\int\frac{dx}{x^2+1}$	1
	$=\frac{1}{2}\log\lvert x-1\rvert-\frac{1}{4}\log\lvert x^2+1\rvert+\frac{1}{2}\tan^{-1}x+C$	1
Q29.	(a) Find a particular solution of the differential equation $\frac{dy}{dx}+y\cot x=4x\operatorname{cosec}x$, $(x\neq 0)$, given that $y=0$ when $x=\frac{\pi}{2}$. **OR** (b) Solve the differential equation $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$.	

A29.	$\frac{dy}{dx}+y.\cot x=4x\operatorname{cosec} x$ $I.F.=e^{\int\cot x\,dx}=e^{\log(\sin x)}=\sin x$ Solution is given by : $y.(\sin x)=\int(4x.\operatorname{cosec} x).\sin x\,dx$ $\Rightarrow y.\sin x=\int 4x\,dx$ $\Rightarrow y.\sin x=2x^2+C$ Now, $y=0,x=\frac{\pi}{2}$ gives $C=-\frac{\pi^2}{2}$ Required solution is : $y.\sin x=2x^2-\frac{\pi^2}{2}$ **OR** $xdy-ydx=\sqrt{x^2+y^2}\,dx$ $\Rightarrow\frac{dy}{dx}=\frac{y+\sqrt{x^2+y^2}}{x}$ ...(1) Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$ ...(2) from (1) and (2), we get $x\frac{dv}{dx}=\sqrt{1+v^2}\Rightarrow\int\frac{dv}{\sqrt{1+v^2}}=\int\frac{dx}{x}$ $\Rightarrow\log\left\|v+\sqrt{1+v^2}\right\|=\log\|x\|+\log C$ $\log\left\|\frac{y}{x}+\sqrt{1+\left(\frac{y}{x}\right)^2}\right\|=\log\|x\|+\log C$ or $y+\sqrt{x^2+y^2}=Cx^2$	 1 ½ 1 ½ ½ ½ ½ 1 ½
Q30.	The objective function z = 4x + 3y of a linear programming problem under some constraints is to be maximized and minimized. The corner points of the feasible region are A(0, 700), B(100, 700), C(200, 600) and D(400, 200). Find the point at which z is maximum and the point at which z is minimum. Also, find the corresponding maximum and minimum values of z.	

A30.	<table><tr><th>Corner Point</th><th>Value of $z=4x+3y$</th></tr><tr><td>$A(0,700)$</td><td>2100</td></tr><tr><td>$B(100,700)$</td><td>2500</td></tr><tr><td>$C(200,600)$</td><td>2600</td></tr><tr><td>$D(400,200)$</td><td>2200</td></tr></table>	2
	$z_{\min}=2100$ at $A(0,700)$ and $z_{\max}=2600$ at $C(200,600)$	1
Q31.	(a) A man is known to speak the truth 3 out of 5 times. He throws a pair of different coins and reports that he got a pair of heads. Find the probability that a pair of heads actually occurs. **OR** (b) From a lot of 10 bulbs which includes 2 defectives, a sample of 2 bulbs is drawn at random without replacement. Find the probability distribution of the number of defective bulbs. Hence, find the mean.	
A31.	E_1 : *getting both heads* E_2 : *Not getting both heads* A : Re*porting two heads*	½
	here, $P(E_1)=\frac{1}{4}, P(E_2)=\frac{3}{4}$ $P(A\|E_1)=\frac{3}{5}, P(A\|E_2)=\frac{2}{5}$	1
	$P(E_1\|A)=\frac{P(E_1).P(A\|E_1)}{P(E_1).P(A\|E_1)+P(E_2).P(A\|E_2)}$ $=\frac{\frac{1}{4}\times\frac{3}{5}}{\frac{1}{4}\times\frac{3}{5}+\frac{3}{4}\times\frac{2}{5}}$	1
	$=\frac{3}{9}$ or $\frac{1}{3}$	½
	OR	

	$P(\text{defective}) = P(D) = \frac{2}{10}, P(\text{non-defective}) = P(N) = \frac{8}{10}$	½
	$X =$ number of defective bulbs; where X can be 0,1 and 2	½
	$P(X=0) = \frac{8}{10} \times \frac{7}{9} = \frac{28}{45}, P(X=1) = \frac{2}{10} \times \frac{8}{9} \times 2 = \frac{16}{45}, P(X=2) = \frac{2}{10} \times \frac{1}{9} = \frac{1}{45}$ Required Probability Distribution is : X: 0, 1, 2 $P(X)$: $\frac{28}{45}$, $\frac{16}{45}$, $\frac{1}{45}$	$1\frac{1}{2}$
	$\text{Mean} = \sum X.P(X) = \frac{18}{45}$ or $\frac{2}{5}$	½

X	0	1	2
$P(X)$	$\frac{28}{45}$	$\frac{16}{45}$	$\frac{1}{45}$

SECTION D

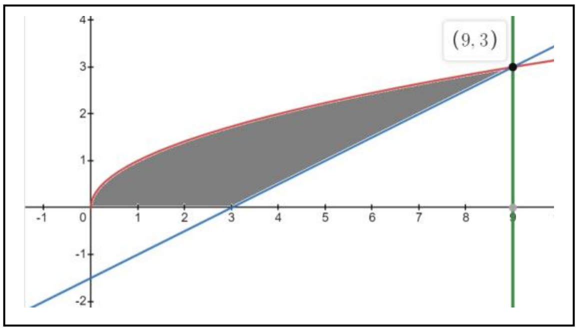
This section comprises long answer (LA) type questions of **5 marks each**.

Q32. (a) A relation R in the set A = {5, 6, 7, 8, 9} is given by R = {(x, y) : |x – y| is divisible by 2}. Write R in roster form and prove that R is an equivalence relation. Also, find the elements related to element 7.

OR

(b) Let A = ℝ – {3} and B = ℝ – {1} be two sets. Prove that the function f : A → B given by f(x) = $\left(\frac{x-2}{x-3}\right)$ is onto. Is the function f one-one ? Justify your answer.

A32.	$R = \{(x, y) : \lvert x - y \rvert \text{ is divisible by } 2\}$ R = {(5, 5), (5, 7), (5, 9), (6, 6), (6, 8), (8, 6), (9, 5), (7, 7), (7, 9), (8, 8), (9, 9), (7, 5), (9, 7)}	1
	For reflexive, as $\lvert x - x \rvert = 0$, is divisible by 2 $\Rightarrow (x, x) \in R$ for all $x \in A \Rightarrow R$ is reflexive.	½
	For symmetric, Let $(x, y) \in R \Rightarrow \lvert x - y \rvert$ is divisible by 2 $\Rightarrow \lvert y - x \rvert$ is divisible by $2 \Rightarrow (y, x) \in R \Rightarrow R$ is symmetric.	1
	For transitive, Let $(x, y) \in R$ and $(y, z) \in R$ $\Rightarrow \lvert x - y \rvert$ is divisible by 2 and $\lvert y - z \rvert$ is divisible by 2. hence $(x - y)$ is divisible by 2 and $(y - z)$ is divisible by 2. $\Rightarrow (x - z)$ is divisible by 2. $\Rightarrow \lvert x - z \rvert$ is divisible by 2 i.e. $(x, z) \in R \Rightarrow R$ is transitive. Since R is reflexive, symmetric and transitive $\Rightarrow R$ is an equivalence relation.	1½
	Here, elements related to $7 = \{5, 7, 9\}$	1

	OR For onto, Let $y = f(x) \Rightarrow y = \dfrac{x-2}{x-3} \Rightarrow x = \dfrac{3y-2}{y-1}$ clearly $y \neq 1 \Rightarrow R_f = R - \{1\}$ i.e. $R_f =$ co-domain $\Rightarrow f$ is onto.	2½
	for one-one Let $x_1, x_2 \in R - \{3\}$ and let $f(x_1) = f(x_2)$ $\Rightarrow \dfrac{x_1 - 2}{x_1 - 3} = \dfrac{x_2 - 2}{x_2 - 3}$ $\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$ $\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.	2½
Q33.	Using matrices, solve the following system of linear equations : $x - y + 2z = 7 \; ; \; 3x + 4y - 5z = -5 \; ; \; 2x - y + 3z = 12$	
A33.	Given system of equations can be written as $AX = B$, where	½
	$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ here, $\lvert A \rvert = 4 \neq 0 \Rightarrow A^{-1}$ exists.	1
	$\therefore \text{adj}A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \Rightarrow A^{-1} = \dfrac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$	2½
	$X = A^{-1}B = \dfrac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} = \dfrac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $\Rightarrow x = 2, y = 1, z = 3$	1
Q34.	Find the area of the region bounded by the curve $y = \sqrt{x}$, the line $x = 2y + 3$ and the x-axis, using integration.	
A34.	Given curves are $y = \sqrt{x}$ and $x = 2y + 3$ Point of intersection is $(9,3)$. 	1
	Required Area $= \int_0^9 \sqrt{x}\, dx - \int_3^9 \left(\dfrac{x-3}{2}\right) dx$	2
	$= \dfrac{2}{3}\left[x^{\frac{3}{2}}\right]_0^9 - \dfrac{1}{4}\left[(x-3)^2\right]_3^9$	1
	$= 9$	1

Q35. (a) Find the shortest distance between the lines whose vector equations are :

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \text{ and}$$

$$\vec{r} = 3\hat{i} - 3\hat{j} - 5\hat{k} + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

OR

(b) Find the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{x-4}{2} = \frac{y}{-6} = \frac{z-1}{3}$. Also, find the perpendicular distance of the given line from the given point.

A35.

$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$, $\vec{a}_2 = 3\hat{i} - 3\hat{j} - 5k$

$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$, $\vec{b}_2 = -2\hat{i} + 3\hat{j} + 8\hat{k}$

here, $\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 5\hat{j} - \hat{k}$ 1

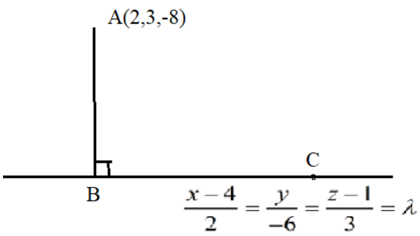
$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix} = 6\hat{i} - 28\hat{j} + 12\hat{k}$ 2

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{964}$ ½

$$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right|$$

$= \left|\frac{(2)(6) + (-5)(-28) + (-1)(12)}{\sqrt{964}}\right| = \frac{140}{\sqrt{964}}$ or $\frac{70}{\sqrt{241}}$ 1+ ½

OR



Any point on line is $(2\lambda + 4, -6\lambda, 3\lambda + 1)$ for some λ. Let $B(2\lambda + 4, -6\lambda, 3\lambda + 1)$ 1

d.r.of $AB = < 2\lambda + 2, -6\lambda - 3, 3\lambda + 9 >$ ½

d.r.of $BC = < 2, -6, 3 >$

$AB \perp BC \Rightarrow 2(2\lambda + 2) - 6(-6\lambda - 3) + 3(3\lambda + 9) = 0$ 1

$\Rightarrow \lambda = -1$ ½

$\therefore B(2, 6, -2)$ 1

Now, $AB = \sqrt{(2-2)^2 + (6-3)^2 + (-2+8)^2} = \sqrt{45}$ or $3\sqrt{5}$ units 1

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36.	**Case Study – 1** A balloon is being inflated with the help of an air pump, and it remains spherical. Its radius, the surface area and the volume of air in it are all increasing. Based on the above, answer the following questions : (i) Are the quantities : radius, surface area and volume of the spherical balloon changing at the same rate or different rates, when air is filled in it ? *1* (ii) Write the expressions for the surface area (S) and the volume (V) of the balloon at any time 't' in terms of radius 'r' at that instant. *1* (iii) (a) At the instant when the radius of the balloon is 6 cm and the radius (r) is increasing at the rate of 2 cm/s, find at what rate the surface area (S) of the balloon is increasing. *2* **OR** (iii) (b) At the instant when the radius of the balloon is 6 cm and the radius (r) is increasing at the rate of 2 cm/s, find at what rate the volume (V) of the spherical balloon is increasing. *2*	
A36.	(i) No, they are changing at the different rates.	1
	$(ii)\, S=4\pi r^2,\ V=\frac{4}{3}\pi r^3$	1
	$(iii)(a)\, S=4\pi r^2 \Rightarrow \frac{dS}{dt}=8\pi r\frac{dr}{dt}$	1
	$\Rightarrow \left.\frac{dS}{dt}\right\vert_{r=6\text{ cm}}=8\pi(6)(2)=96\pi\text{ cm}^2/\text{s}$	1
	OR	
	$(b)\, V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$	1
	$\Rightarrow \left.\frac{dV}{dt}\right\vert_{r=6\text{ cm}}=4\pi(6)^2(2)=288\pi\text{ cm}^3/\text{s}$	1

Q37.	**Case Study – 2** A fighter-jet of the enemy is flying along the parabolic path $4y = x^2$. A soldier is located at the point (0, 5) and is aiming to shoot down the jet when it is nearest to him. Based on the above, answer the following questions : (i) Let (x, y) be the position of the jet at any instant. Express the distance between the soldier and the jet as the function f(x). *1* (ii) Taking $S = [f(x)]^2$, find $\frac{dS}{dx}$. *1* (iii) (a) What will be the position of the jet when the soldier shoots it down ? *2* **OR** (iii) (b) What will be the distance between the soldier and the jet at the instant when he shoots it down ? *2*	
A37.	$(i)\ \text{Distance} = \sqrt{(x-0)^2 + \left(\frac{x^2}{4} - 5\right)^2} = f(x)$	1
	$(ii)\ S = [f(x)]^2 = x^2 + \left(\frac{x^2}{4} - 5\right)^2$ $\Rightarrow \frac{dS}{dx} = 2x + 2\left(\frac{x^2}{4} - 5\right)\left(\frac{x}{2}\right) = \frac{1}{4}x\left(x^2 - 12\right)$	1
	$(iii)(a)\ \frac{dS}{dx} = 0 \Rightarrow x = 0, \pm 2\sqrt{3}$	½
	Showing $\left(2\sqrt{3}, 3\right)$ and $\left(-2\sqrt{3}, 3\right)$ are the point of minima.	1
	$\therefore$ Required position is $\left(\pm 2\sqrt{3}, 3\right)$.	½
	OR	
	(b) getting $x = \pm 2\sqrt{3}$ as a point of minima,	1½
	$\text{Distance} = \sqrt{\left(\pm 2\sqrt{3}\right)^2 + (3-5)^2} = 4$ units	½

Q38.	Read the following passage and answer the questions given below : There are ten cards numbered 1 to 10 and they are placed in a box and then mixed up thoroughly. Then one card is drawn at random from the box. Based on the above, answer the following questions : (i) What is the probability that the number on the drawn card is greater than 4 ? *2* (ii) If it is known that the number on the drawn card is greater than 4, then what is the probability that it is an even number ? *2*	
A38.	$S=\{1,2,3,...,9,10\}$ $(i)\,P(\text{getting number}>4)$ $=P(5\,\text{or}\,6\,\text{or}\,7\,\text{or}\,8\,\text{or}\,9\,\text{or}\,10)$ $=\frac{6}{10}\text{ or }\frac{3}{5}$ $(ii)\,A:\text{getting even number}=\{2,4,6,8,10\}$ $B:\text{getting number greater than }4=\{5,6,7,8,9,10\}$ $A\cap B=\{6,8,10\}$ Now $P(A\mid B)=\frac{P(A\cap B)}{P(B)}$ $=\frac{\frac{3}{10}}{\frac{6}{10}}=\frac{1}{2}$	 1 1 ½ ½ 1