Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Certificate Examination, 2023
MATHEMATICS PAPER CODE 65/4/3

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives the impression that answer is correct, and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**

11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
14	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

Senior Secondary School Examination, 2022-23

MATHEMATICS(Subject Code-041)

(Paper Code-65/4/3)

Section A

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	Marks
	SECTION A Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of **1 mark each.**	
1.	If A is a 3×4 matrix and B is a matrix such that $A'B$ and AB' are both defined, then the order of the matrix B is : (a) 3×4 (b) 3×3 (c) 4×4 (d) 4×3	
Sol.	(a)3 x 4	1
2.	If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq units, then k is (a) 12 (b) -2 (c) $-12, -2$ (d) $12, -2$	
Sol.	(d)12, – 2	1
3.	If $f(x) = 2\|x\| + 3\|\sin x\| + 6$, then the right hand derivative of f(x) at x = 0 is : (a) 6 (b) 5 (c) 3 (d) 2	
Sol.	(b)5	1

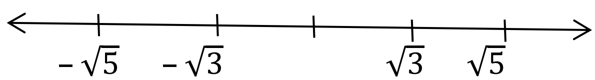
4.	If $x\begin{bmatrix}1\\2\end{bmatrix} + y\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}4\\9\end{bmatrix}$, then : (a) $x = 1,\ y = 2$ (b) $x = 2,\ y = 1$ (c) $x = 1,\ y = -1$ (d) $x = 3,\ y = 2$	
Sol.	(b) $x = 2,\ y = 1$	1
5.	If a matrix A = [1 2 3], then the matrix AA′ (where A′ is the transpose of A) is : (a) 14 (b) $\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$ (c) $\begin{bmatrix}1&2&3\\2&3&1\\3&1&2\end{bmatrix}$ (d) [14]	
Sol.	(d) [14]	1
6.	The product $\begin{bmatrix}a&b\\-b&a\end{bmatrix}\begin{bmatrix}a&-b\\b&a\end{bmatrix}$ is equal to : (a) $\begin{bmatrix}a^2+b^2&0\\0&a^2+b^2\end{bmatrix}$ (b) $\begin{bmatrix}(a+b)^2&0\\(a+b)^2&0\end{bmatrix}$ (c) $\begin{bmatrix}a^2+b^2&0\\a^2+b^2&0\end{bmatrix}$ (d) $\begin{bmatrix}a&0\\0&b\end{bmatrix}$	
Sol.	(a) $\begin{bmatrix}\boldsymbol{a^2+b^2}&\mathbf{0}\\\mathbf{0}&\boldsymbol{a^2+b^2}\end{bmatrix}$	1
7.	Distance of the point (p, q, r) from y-axis is : (a) q (b) $\lvert q\rvert$ (c) $\lvert q\rvert + \lvert r\rvert$ (d) $\sqrt{p^2+r^2}$	
Sol.	(d) $\sqrt{\boldsymbol{p^2 + r^2}}$	1

8.	The solution set of the inequation $3x + 5y < 7$ is : (a) whole xy-plane except the points lying on the line $3x + 5y = 7$. (b) whole xy-plane along with the points lying on the line $3x + 5y = 7$. (c) open half plane containing the origin except the points of line $3x + 5y = 7$. (d) open half plane not containing the origin.	
Sol.	(c) open half plane containing the origin except the points of line $3x + 5y=7$	1
9.	If $\int_0^a 3x^2\,dx = 8$, then the value of 'a' is : (a) 2 (b) 4 (c) 8 (d) 10	
Sol.	(a)2	1
10.	The sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 2\hat{k}$ is : (a) $\sqrt{\frac{5}{21}}$ (b) $\frac{5}{\sqrt{21}}$ (c) $\sqrt{\frac{3}{21}}$ (d) $\frac{4}{\sqrt{21}}$	
Sol.	(a) $\sqrt{\frac{5}{21}}$	1
11.	The order and degree (if defined) of the differential equation, $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 = x \sin\left(\frac{dy}{dx}\right)$ respectively are : (a) 2, 2 (b) 1, 3 (c) 2, 3 (d) 2, degree not defined	

Sol.	(d)2, degree not defined	1
12.	$\int e^{5 \log x} dx$ is equal to : (a) $\frac{x^5}{5} + C$ (b) $\frac{x^6}{6} + C$ (c) $5x^4 + C$ (d) $6x^5 + C$	
Sol.	(b)$\frac{x^6}{6} + C$	1
13.	A unit vector along the vector $4\hat{i} - 3\hat{k}$ is : (a) $\frac{1}{7}(4\hat{i} - 3\hat{k})$ (b) $\frac{1}{5}(4\hat{i} - 3\hat{k})$ (c) $\frac{1}{\sqrt{7}}(4\hat{i} - 3\hat{k})$ (d) $\frac{1}{\sqrt{5}}(4\hat{i} - 3\hat{k})$	
Sol.	(b)$\frac{1}{5}(4\hat{i} - 3\hat{k})$	1
14.	Which of the following points satisfies both the inequations $2x + y \leq 10$ and $x + 2y \geq 8$? (a) $(-2, 4)$ (b) $(3, 2)$ (c) $(-5, 6)$ (d) $(4, 2)$	
Sol.	(d)(4, 2)	1

15.	If $y = \sin^2(x^3)$, then $\frac{dy}{dx}$ is equal to : (a) $2 \sin x^3 \cos x^3$ (b) $3x^3 \sin x^3 \cos x^3$ (c) $6x^2 \sin x^3 \cos x^3$ (d) $2x^2 \sin^2(x^3)$	
Sol.	(c)$6x^2 \sin(x^3) \cos(x^3)$	1
16.	The point (x, y, 0) on the xy-plane divides the line segment joining the points (1, 2, 3) and (3, 2, 1) in the ratio : (a) 1 : 2 internally (b) 2 : 1 internally (c) 3 : 1 internally (d) 3 : 1 externally	
Sol.	(d)3 : 1 externally	1
17.	The events E and F are independent. If P(E) = 0·3 and $P(E \cup F) = 0{\cdot}5$, then P(E/F) – P(F/E) equals : (a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{35}$ (d) $\frac{1}{70}$	
Sol.	(d)$\frac{1}{70}$	1
18.	The integrating factor for solving the differential equation $x\frac{dy}{dx} - y = 2x^2$ is : (a) e^{-y} (b) e^{-x} (c) x (d) $\frac{1}{x}$	
Sol.	(d)$\frac{1}{x}$	1

	*Questions number **19** and **20** are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.* (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (c) Assertion (A) is true and Reason (R) is false. (d) Assertion (A) is false and Reason (R) is true.	
19.	*Assertion (A) :* The lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ are perpendicular, when $\vec{b}_1 \cdot \vec{b}_2 = 0$. *Reason (R) :* The angle θ between the lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is given by $\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{\lvert \vec{b}_1 \rvert \lvert \vec{b}_2 \rvert}$	
Sol.	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)	1
20.	*Assertion (A) :* All trigonometric functions have their inverses over their respective domains. *Reason (R) :* The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.	
Sol.	(d)Assertion (A) is false and Reason (R) *is true*.	1
	SECTION B This section comprises very short answer (VSA) type questions of 2 marks each.	
21.	If $xy = e^{x-y}$, then show that $\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$.	
Sol.	Given $xy = e^{x-y}$, gives $x - y = \log x + \log y$	$\frac{1}{2}$

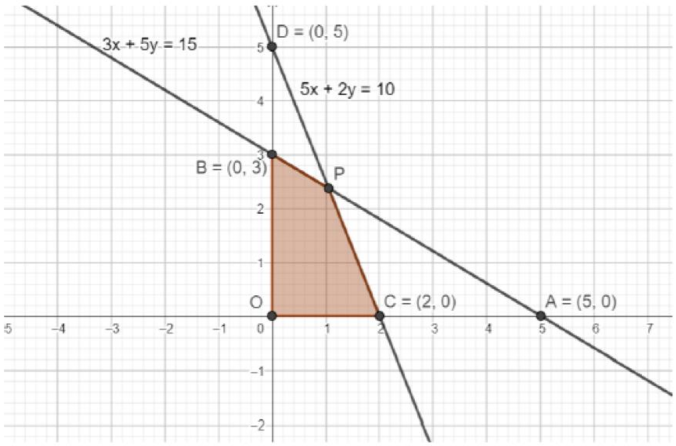
	$\Rightarrow 1 - \frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}\frac{dy}{dx}$ $\Rightarrow \left(\frac{1}{y} + 1\right)\frac{dy}{dx} = 1 - \frac{1}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{x-1}{x} \times \frac{y}{1+y} = \frac{y(x-1)}{x(1+y)}$	1 $\frac{1}{2}$
22.	(a) Find the domain of $y = \sin^{-1}(x^2 - 4)$. **OR** (b) Evaluate : $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$	
Sol.	(a) Domain of $\sin^{-1} x$ is $-1 \le x \le 1$ $\therefore -1 \le x^2 - 4 \le 1$ or $x^2 \ge 3, x^2 \le 5$ $\Rightarrow x \ge \sqrt{3}$ or $x \le -\sqrt{3}$, $x \le \sqrt{5}$ or $x \ge -\sqrt{5}$  $\therefore$ Domain is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$ **OR** (b) $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right] = \cos^{-1}\left[\cos\left(\frac{7\pi}{3}\right)\right]$ $= \cos^{-1}\left[\cos\left(2\pi + \frac{\pi}{3}\right)\right]$	**$\frac{1}{2}$** 1 **$\frac{1}{2}$** 1 **$\frac{1}{2}$**

	$= \cos^{-1}\left[\cos\frac{\pi}{3}\right] = \frac{\pi}{3}.$	**$\frac{1}{2}$**
23.	If the projection of the vector $\hat{i} + \hat{j} + \hat{k}$ on the vector $p\hat{i} + \hat{j} - 2\hat{k}$ is $\frac{1}{3}$, then find the value(s) of p.	
Sol.	Here, $\frac{(\hat{i} + \hat{j} + \hat{k}) \cdot (p\hat{i} + \hat{j} - 2\hat{k})}{\sqrt{p^2 + 1 + 4}} = \frac{1}{3}$ $\Rightarrow \frac{p - 1}{\sqrt{p^2 + 5}} = \frac{1}{3}$ $\Rightarrow 8p^2 - 18p + 4 = 0$ $4p^2 - 9p + 2 = 0$ $4p^2 - 8p - p + 2 = 0$ $(4p - 1)(p - 2) = 0$ $\Rightarrow p = 2$ or $p = \frac{1}{4}$	$\frac{1}{2}$ $\frac{1}{2}$ 1
24.	Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate.	
Sol.	Here, $\frac{dx}{dt} = \frac{dy}{dt}$ Given $y^2 = 8x$ gives $2y. \frac{dy}{dt} = 8\frac{dx}{dt}$ $\Rightarrow 2y = 8$ or $y = 4$ Also, $y = 4$ gives $x = 2$. Thus, the point (2, 4)	$\frac{1}{2}$ 1 $\frac{1}{2}$

25.	(a) Find the vector equation of the line passing through the point (2, 1, 3) and perpendicular to both the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$; $\frac{x}{-3}=\frac{y}{2}=\frac{z}{5}$. **OR** (b) The equations of a line are 5x – 3 = 15y + 7 = 3 – 10z. Write the direction cosines of the line and find the coordinates of a point through which it passes.	
Sol.	(a)Vector equation of the line passing through (2, 1, 3) is $\vec{r} = (2\hat{\imath}+\hat{\jmath}+3\hat{k})+\lambda(a\hat{\imath}+b\hat{\jmath}+c\hat{k})$	$\frac{1}{2}$
	Line $\vec{r}$ is perpendicular to the given lines then a + 2b + 3c = 0; – 3a + 2b + 5c = 0 $\Rightarrow \frac{a}{4}=\frac{b}{-14}=\frac{c}{8}$=k'(say) $\Rightarrow$ a = 2k, b = – 7k and c = 4k	1
	Thus, the required vector equation is $\vec{r} = (2\hat{\imath}+\hat{\jmath}+3\hat{k})+\lambda\,(2\hat{\imath}-7\hat{\jmath}+4\hat{k})$	$\frac{1}{2}$
	OR (b) The equation of the given line is $\frac{x-3/5}{1/5}=\frac{y+7/15}{1/15}=\frac{z-3/10}{-1/10}$	1
	Its direction ratios are $\langle\frac{1}{5},\frac{1}{15},-\frac{1}{10}\rangle$or $\langle 6, 2, -3\rangle$ Direction cosines are $\langle\pm\frac{6}{7},\pm\frac{2}{7},\mp\frac{3}{7}\rangle$	$\frac{1}{2}$

	The point through which it passes is $\left(\frac{3}{5}, \frac{-7}{15}, \frac{3}{10}\right)$	$\frac{1}{2}$
	SECTION C This section comprises short answer (SA) type questions of 3 marks each.	
26.	**Find :** $$\int \frac{2}{(1-x)(1+x^2)}\,dx$$	
Sol.	Let $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$ $\Rightarrow$ A = 1, B = 1, C = 1 Hence, $I = \int \frac{2}{(1-x)(1-x^2)}\,dx = \int \left[\frac{1}{1-x} + \frac{x+1}{1+x^2}\right] dx$ $= \int \frac{1}{1-x}\,dx + \int \frac{x+1}{x^2+1}\,dx$ $= \int \frac{1}{1-x}\,dx + \frac{1}{2}\int \frac{2x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$ $= -\log\lvert 1-x\rvert + \frac{1}{2}\log(x^2+1) + \tan^{-1}(x) + C$	1 1 1
27.	(a) **Evaluate :** $$\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}\,dx$$ **OR** (b) **Evaluate :** $$\int_{1}^{3} \{\lvert(x-1)\rvert + \lvert(x-2)\rvert\}\,dx$$	
Sol.	(a) $I = \int_{1/3}^{1} \frac{(x-x^3)^{\frac{1}{3}}}{x^4}\,dx$	

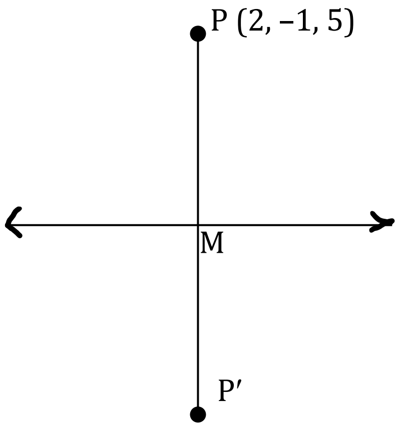
	$= \int_{1/3}^{1} \frac{(x^3)^{1/3}\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^4}\, dx = \int_{1/3}^{1} \frac{\left(\frac{1}{x^2} - 1\right)^{1/3}}{x^3}\, dx$	1
	Put $\left(\frac{1}{x^2} - 1\right) = t$ so that $\frac{-2}{x^3} dx = dt$	$\frac{1}{2}$
	Thus,	
	$I = \int_0^8 t^{\frac{1}{3}} \times \frac{dt}{(2)}$	$\frac{1}{2}$
	$= \frac{48}{8} = 6.$	1
	OR	
	(b) $\int_1^3 (\lvert x-1 \rvert + \lvert x-2 \rvert)\, dx$	
	$= \int_1^3 (x-1)\, dx - \int_1^2 (x-2)\, dx + \int_2^3 (x-2)\, dx$	$1\frac{1}{2}$
	$= \left\lvert\frac{(x-1)^2}{2}\right\rvert_1^3 - \left\lvert\frac{(x-2)^2}{2}\right\rvert_1^2 + \left\lvert\frac{(x-2)^2}{2}\right\rvert_2^3$	$\frac{1}{2}$
	$= (2) - \left(0 - \frac{1}{2}\right) + \frac{1}{2} = 3$	1
28.	Solve the following linear programming problem graphically : Maximise z = 5x + 3y subject to the constraints $3x + 5y \le 15,$ $5x + 2y \le 10,$ $x, y \ge 0.$	

Sol.	 | Corner Points | Value of Z | | O(0,0) | 0 | | B(0,3) | 9 | | C(2,0) | 10 | | P(20/19,45/19) | $\frac{235}{19}$ → Max |	2 for correct graph 1
29.	**From a lot of 30 bulbs which include 6 defective bulbs, a sample of 2 bulbs is drawn at random one by one with replacement. Find the probability distribution of the number of defective bulbs and hence find the mean number of defective bulbs.**	
Sol.	Let X be the random variable which denotes the number of defective bulbs in a sample of 2 bulbs drawn. Here X may take values 0, 1 or 2. Let A and B be the events of drawing a defective bulb and non defective bulb respectively $P(A) = \frac{6}{30} = \frac{1}{5}$; $P(B) = 1 - \frac{1}{5} = \frac{4}{5}$ Now $P(X = 0) = \frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$	$\frac{1}{2}$ $\frac{1}{2}$

	$P(X=1) = 2\left[\frac{4}{5} \times \frac{1}{5}\right] = \frac{8}{25}$ $P(X=2) = \frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$ Thus P.D. of X is <table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(X)</td><td>$\frac{16}{25}$</td><td>$\frac{8}{25}$</td><td>$\frac{1}{25}$</td></tr><tr><td>XP(X)</td><td>0</td><td>$\frac{8}{25}$</td><td>$\frac{2}{25}$</td></tr></table>Mean= $\sum X\,P(X) = \frac{10}{25} = \frac{2}{5}$	$1\frac{1}{2}$ $\frac{1}{2}$
30.	(a) Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$, $y(1) = 0$. **OR** (b) Find the general solution of the differential equation $e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0$.	
Sol.	(a) $\frac{dy}{dx} = \frac{x+y}{x} \Rightarrow \frac{dy}{dx} = 1 + \frac{y}{x}$ Let $\frac{y}{x} = v$. Then $x\frac{dv}{dx} + v = \frac{dy}{dx}$ So, D.E. becomes $x\frac{dv}{dx} + v = 1 + v$ $\Rightarrow dv = \frac{dx}{x}$ $\Rightarrow v = \log\|x\| + c$ $\Rightarrow y = x\log\|x\| + cx$ $\Rightarrow x = 1, y = 0 \Rightarrow c = 0 \Longrightarrow y = x\log\|x\|$	1 1 1

	[can also be solved ,taking as first order linear diff. eqn]	
	OR	
	(b) The given D.E. is $\frac{\sec^2 y}{\tan y}dy = -\frac{e^x}{1-e^x}dx$	1
	Integrating $\Rightarrow \log \lvert \tan y \rvert = \log \lvert 1 - e^x \rvert + \log C$	$1\frac{1}{2}$
	$\Rightarrow \tan y = C(1 - e^x)$	$\frac{1}{2}$
31.	(a) Evaluate : $\int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$ **OR** (b) Evaluate : $\int_{-2}^{2} \frac{x^2}{1+5^x}dx$	
Sol.	(a) $I = \int_{\pi/4}^{\pi/2} e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$	
	Put 2x = t so that 2 dx = dt	$\frac{1}{2}$
	When $x = \frac{\pi}{2}$ $t = \pi$, $x = \frac{\pi}{4}$ $t = \frac{\pi}{2}$	
	Thus	
	$I = \int_{\pi/2}^{\pi} e^t\left(\frac{1-\sin t}{1-\cos t}\right)\frac{dt}{2}$	
	$= \int_{\pi/2}^{\pi} e^t\left(\frac{1-2\sin t/2\cos t/2}{2\sin^2 t/2}\right)\frac{dt}{2}$	1
	$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} e^t\left(\frac{1}{2}cosec^2\frac{t}{2} - \cot\frac{t}{2}\right)dt$	$\frac{1}{2}$

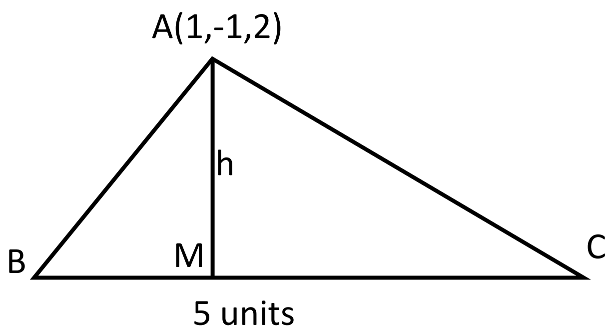
	$= -\frac{1}{2}\left| e^t \cot \frac{t}{2} \right|_{\pi/2}^{\pi}$ $= -\frac{1}{2}\left| e^{\pi} \cot \frac{\pi}{2} - e^{\pi/2} \cot \frac{\pi}{4} \right|$ $= \frac{1}{2} e^{\pi/2}$. **OR** (b) $I = \int_{-2}^{2} \frac{x^2}{1+5^x}$ dx ----------- (1) Replacing x by -2+2-x $= \int_{-2}^{2} \frac{(-2+2-x)^2}{1+5^{(-2+2-x)}}$ dx I= $\int_{-2}^{2} \frac{x^2 5^x}{1+5^x}$ dx -------------(2) Adding (1) + (2) $2I = \int_{-2}^{2} \frac{x^2(1+5^x)}{1+5^x} dx = \int_{-2}^{2} x^2 \ dx = \left|\frac{x^3}{3}\right|_{-2}^{2} = \frac{8}{3} + \frac{8}{3} = \frac{16}{3}$ $\Rightarrow I = \frac{16}{6}$ or $\frac{8}{3}$	$\frac{1}{2}$ $\frac{1}{2}$ 1 $\frac{1}{2}$ 1 $\frac{1}{2}$
	SECTION D This section comprises of Long Answer (LA) type questions of 5 marks each.	
32.	(a) Find the image of the point (2, −1, 5) in the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. **OR** (b) Vertices B and C of Δ ABC lie on the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4}$. Find the area of Δ ABC given that point A has coordinates (1, −1, 2) and the line segment BC has length of 5 units.	
Sol.	The given line is $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$(say)	



	Let the given point be P(2, –1, 5) Coordinates of M are $(10\lambda + 11, -4\lambda - 2, -11\lambda - 8)$ where M is the foot of perpendicular from P. Direction ratios of PM are $(10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$ As given line is perpendicular to PM, we have $10(10\lambda + 9) - 4(-4\lambda - 1) - 11(-11\lambda - 13) = 0$ $\Rightarrow \lambda = -1$ $\Rightarrow M(1, 2, 3)$ Let P' (α, β, γ) be the image of the point P such that M is the mid-point of PP'. $\frac{\alpha + 2}{2} = 1; \frac{\beta - 1}{2} = 2; \frac{\gamma + 5}{2} = 3;$ $\Rightarrow \alpha = 0, \beta = 5, \gamma = 1$ Hence, image of P(2, – 1, 5) is P'(0, 5, 1)	1 1+1 1 1

OR

Let h be the height of Δ ABC. Then, h is the length of the perpendicular from A(1, – 1, 2) to the line $\frac{x+2}{2} = \frac{y-1}{1} = \frac{z}{4} = \lambda$(say)



General point M of the line is $(2\lambda - 2, \lambda + 1, 4\lambda)$ | 1

Direction ratios of AM are $(2\lambda - 3, \lambda + 2, 4\lambda - 2)$

1

Since AM $\perp$ BC

$2(2\lambda - 3) + 1(\lambda + 2) + 4(4\lambda - 2) = 0$

$\Rightarrow \lambda = \frac{4}{7}$ 1

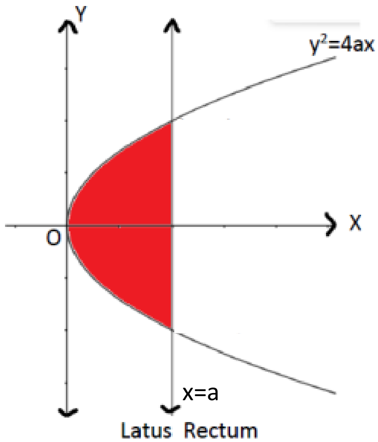
Hence, $M\left(-\frac{6}{7}, \frac{11}{7}, \frac{16}{7}\right)$ 1

$\Rightarrow h = \sqrt{\left(-\frac{6}{7} - 1\right)^2 + \left(\frac{11}{7} + 1\right)^2 + \left(\frac{16}{7} - 2\right)^2}$

$= \frac{\sqrt{497}}{7}$

Area of ABC $= \frac{1}{2} \times 5 \times \frac{\sqrt{497}}{7}$

	$= \frac{5\sqrt{497}}{14}$ sq. units.	1
33.	Find the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$. Using the inverse, A^{-1}, solve the system of linear equations $x - y + 2z = 1;\ 2y - 3z = 1;\ 3x - 2y + 4z = 3.$	
Sol.	$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$	
	$\|A\| = 1(8 - 6) + 1(0 + 9) + 2(0 - 6) = -1 \neq 0$	1
	$\therefore$ A is invertible.	
	$\text{adj } A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$	$1\frac{1}{2}$
	$\Rightarrow A^{-1} = \frac{1}{\|A\|} \text{adj } A = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$	$\frac{1}{2}$
	The given system of equation can be written as AX = B, when	
	$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$	
	$\Rightarrow X = A^{-1} B$	1
	$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$	1

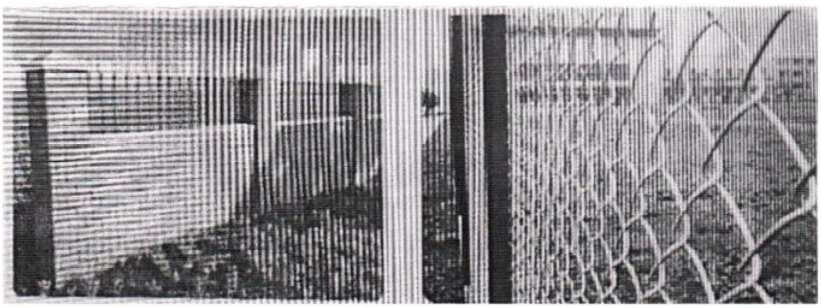
	$\Rightarrow x = 1, y = 2, z = 1$	
34.	Using integration, find the area of the region bounded by the parabola $y^2 = 4ax$ and its latus rectum.	
Sol.		1
	Required area $= 2\int_0^a 2\sqrt{ax}\ dx$	$1\frac{1}{2}$
	$= 4\sqrt{a}\int_0^a \sqrt{x}\ dx$	$\frac{1}{2}$
	$= 4\sqrt{a}\left[\frac{2x^{3/2}}{3}\right]_0^a$	1
	$= \frac{8}{3}a\sqrt{a}\ \sqrt{a}$	
	$= \frac{8}{3}a^2$	1
35.	(a) If N denotes the set of all natural numbers and R is the relation on N × N defined by (a, b) R (c, d), if ad(b + c) = bc(a + d). Show that R is an equivalence relation. **OR** (b) Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.	

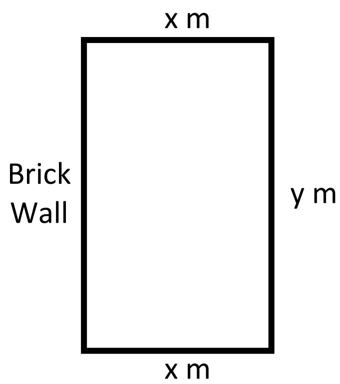
Sol.	(a)Reflexive :Here, (a, b) R (a, b) $\forall (a, b) \in N\ X\ N$ since ab(b + a) = ba(a + b) is always true. *Symmetric* :Let (a, b) R (c, d) $\forall (a, b), (c, d) \in NX\ N$. Then, ad(b + c) = bc(a + d) $\Rightarrow$ bc(a + d) = ad(b + c) $\Rightarrow$ (c, d) R (a, b) *Transitive* : Let (a, b) R (c, d) and (c, d) R (e, f) $\forall (a, b), (c, d), (e, f) \in NX\ N$. Then ad (b + c) = bc(a + d) and cf(d + e) = de(c + f) $\Rightarrow \frac{b+c}{bc} = \frac{a+d}{ad}$ and $\frac{d+e}{de} = \frac{c+f}{cf}$ $\frac{1}{c} + \frac{1}{b} = \frac{1}{d} + \frac{1}{a} \Rightarrow \frac{1}{e} + \frac{1}{d} = \frac{1}{f} + \frac{1}{c}$ Adding, we get $\frac{1}{c} + \frac{1}{b} + \frac{1}{e} + \frac{1}{d} = \frac{1}{d} + \frac{1}{a} + \frac{1}{f} + \frac{1}{c}$ $\Rightarrow \frac{1}{b} + \frac{1}{e} = \frac{1}{a} + \frac{1}{f}$ $\Rightarrow \frac{e+b}{be} = \frac{f+a}{af}$ $\Rightarrow$ af(b + e) = be(a + f) $\Rightarrow$ (a, b) R (e, f) Hence, R is equivalence relation. **OR** (b) For one-one Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$	1 $1\frac{1}{2}$ for symmetric 2 for transitive $\frac{1}{2}$

	$\Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$ $\Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$ $\Rightarrow 16x_1 = 16x_2$ $\Rightarrow x_1 = x_2$ $\Rightarrow$ f is one-one For onto let $y \in R$, and for some x. Let $y = \frac{4x}{3x+4}$ $\Rightarrow 3xy + 4y = 4x$ $\Rightarrow x(3y - 4) = -4y$ $\Rightarrow x = -\frac{4y}{3y-4}$ or $x = \frac{4y}{4-3y}$ x is real if $y \neq \frac{4}{3}$. So $R_f = R - \left\{\frac{4}{3}\right\} \neq Codomain\ (f)$ So, f is not onto.	2 1 1 1
	SECTION E This section comprises of 3 case-study based questions of 4 marks each.	

36.	A building contractor undertakes a job to construct 4 flats on a plot along with parking area. Due to strike the probability of many construction workers not being present for the job is 0·65. The probability that many are not present and still the work gets completed on time is 0·35. The probability that work will be completed on time when all workers are present is 0·80. Let : E_1 : represent the event when many workers were not present for the job; E_2 : represent the event when all workers were present; and E : represent completing the construction work on time. Based on the above information, answer the following questions : (i) What is the probability that all the workers are present for the job ? *1* (ii) What is the probability that construction will be completed on time ? *1* (iii) (a) What is the probability that many workers are not present given that the construction work is completed on time ? *2* **OR** (iii) (b) What is the probability that all workers were present given that the construction job was completed on time ? *2*	
Sol.	(i) $P(E_2) = 1 - P(E_1) = 1 - 0{\cdot}65 = 0{\cdot}35$ (ii) $P(E) = P(E_1) \,.\, P\left(\frac{E}{E_1}\right) + P(E_2) \,.\, P\left(\frac{E}{E_2}\right)$ $= 0{\cdot}65 \times 0{\cdot}35 + 0{\cdot}35 \times 0{\cdot}8$ $= 0{\cdot}35 \times 1{\cdot}45$ $= 0{\cdot}51$ (iii) (a) $P\left(\frac{E_1}{E}\right) = \frac{P(E_1)\ .\ P(E/E_1)}{P(E_1).P(E/E_1)+P(E_2).P(E/E_2)} = \frac{0{\cdot}65 \times 0{\cdot}35}{0{\cdot}51} = 0{\cdot}45$	1 1 2

	OR (iii) (b) $P\left(\frac{E_2}{E}\right) = \frac{P(E_2)\ .\ P(E/E_2)}{P(E_1).P(E/E_1)+P(E_2).P(E/E_2)} = \frac{0{\cdot}35 \times 0{\cdot}8}{0{\cdot}51} = 0{\cdot}55$	2
37.	Let f(x) be a real valued function. Then its • Left Hand Derivative (L.H.D.) : $Lf'(a) = \lim_{h\to 0} \frac{f(a-h)-f(a)}{-h}$ • Right Hand Derivative (R.H.D.) : $Rf'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ Also, a function f(x) is said to be differentiable at x = a if its L.H.D. and R.H.D. at x = a exist and both are equal. For the function $f(x) = \begin{cases} \lvert x-3 \rvert, & x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ answer the following questions : (i) What is R.H.D. of f(x) at x = 1 ? *1* (ii) What is L.H.D. of f(x) at x = 1 ? *1* (iii) (a) Check if the function f(x) is differentiable at x = 1. *2* **OR** (iii) (b) Find the f′(2) and f′(– 1). *2*	
Sol.	(i)R.H.D. of f (x) at x = 1 = $\lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$ $= \lim_{h\to 0} \frac{\lvert 1+h-3 \rvert - \lvert -2 \rvert}{h} = \lim_{h\to 0} \frac{2-h-2}{h} = -1$ (ii) L.H.D. of f (x) at x = 1 = $\lim_{h\to 0} \frac{f(1-h)-f(1)}{-h}$ $= \lim_{h\to 0} \frac{-1}{h}\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]$ $= \lim_{h\to 0} \left[\frac{h^2 - 2h + 1 - 6 + 6h + 13 - 8}{-4h}\right]$ $= \lim_{h\to 0} \left[\frac{h^2 + 4h}{-4h}\right] = -1.$ (iii) (a) Since L.H.D. of f(x) at x = 1	1 1

	= R.H.D. of f(x) at x = 1, f(x) is differentiable at x = 1. **OR** (iii) (b) $f(x) = \begin{cases} x - 3, & x \geq 3 \\ 3 - x, & 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & x < 1 \end{cases}$ $[f'(x)]_{x=2} = 0 - 1 = -1$ $[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2} = -2$	2 1 1
38.	Sooraj's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in the figure. He has 200 metres of fencing wire.  Based on the above information, answer the following questions : (i) Let 'x' metres denote the length of the side of the garden perpendicular to the brick wall and 'y' metres denote the length of the side parallel to the brick wall. Determine the relation representing the total length of fencing wire and also write A(x), the area of the garden. *2* (ii) Determine the maximum value of A(x). *2*	
Sol.	(i) (a) 2x + y = 200 (b) A(x) = xy=x(200-x)	1 1



	(ii) From (a) and (b) of (1) we have $A(x) = x(200 - 2x)$ $= 200x - 2x^2$ From max./min of A(x) $\frac{dA}{dx} = 0$ i.e. $200 - 4x = 0$ $\Rightarrow x = 50.$ $\frac{d^2A}{dx^2} = -4$ $\left(\frac{d^2A}{dx^2}\right)_{x=50} < 0$ Hence, A(x) is maximum at x = 50 Thus, Max $A(x) = 200(50) - 2(50)^2$ $= 10000 - 5000$ $= 5000$ sqm.	 1 1