Marking Scheme **Strictly Confidential** **(For Internal and Restricted use only)** **Senior School Certificate Examination, 2023** **MATHEMATICS PAPER CODE 65/2/1**	
<u>General Instructions: -</u>	
1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them.**
4	The Marking scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X" be marked. Evaluators will not put right (✓) while evaluating which gives the impression that answer is correct, and no marks are awarded. **This is most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	**<u>In Q1-Q20, if a candidate attempts the question more than once (without canceling the previous attempt), marks shall be awarded for the first attempt only and the other answer scored out with a note "Extra Question".</u>**
10	**<u>In Q21-Q38, if a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".</u>**
11	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
12	A full scale of marks __________ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) must be used. Please do not hesitate to award full marks if the answer deserves it.
13	Every examiner has to necessarily do evaluation work for full working hours i.e., 8 hours every day and evaluate 20 answer books per day (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.

14	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - ● Leaving answer or part thereof unassessed in an answer book. ● Giving more marks for an answer than assigned to it. ● Wrong totaling of marks awarded on an answer. ● Wrong transfer of marks from the inside pages of the answer book to the title page. ● Wrong question wise totaling on the title page. ● Wrong totaling of marks of the two columns on the title page. ● Wrong grand total. ● Marks in words and figures not tallying/not same. ● Wrong transfer of marks from the answer book to online award list. ● Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) ● Half or a part of answer marked correct and the rest as wrong, but no marks awarded.
15	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
16	Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
17	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for spot Evaluation**" before starting the actual evaluation.
18	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
19	The candidates are entitled to obtain photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/2/1

EXPECTED ANSWER/VALUE POINTS

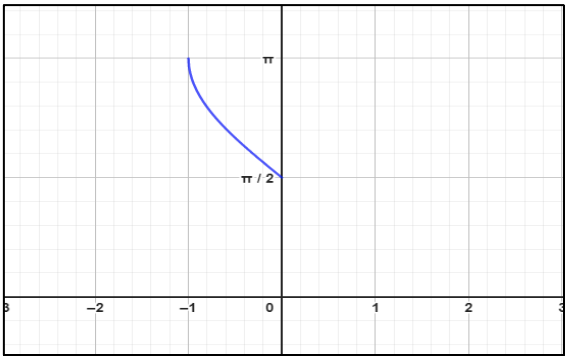
SECTION A

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A (Question nos. 1 to 18 are Multiple choice Questions carrying 1 mark each)	
1.	If $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then A^{2023} is equal to (A) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & 2023 \\ 0 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2023 & 0 \\ 0 & 2023 \end{bmatrix}$	
Ans	**(C)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	**1**
2.	If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$, where P is a symmetric and Q is a skew symmetric matrix, then Q is equal to (A) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 5/2 \\ -5/2 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & -5/2 \\ 5/2 & 4 \end{bmatrix}$	
Ans	**(B)** $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$	**1**
3.	If $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & a & 1 \end{bmatrix}$ is non-singular matrix and $a \in A$, then the set A is (A) $\mathbb{R}$ (B) $\{0\}$ (C) $\{4\}$ (D) $\mathbb{R} - \{4\}$	
Ans	**(D)** $\mathbb{R} - \{4\}$	**1**
4.	If $\lvert A \rvert = \lvert kA \rvert$, where A is a square matrix of order 2, then sum of all possible values of k is (A) 1 (B) –1 (C) 2 (D) 0	

Ans	**(D) 0**	**1**
5.	If $\frac{d}{dx}$ [f (x)] = ax + b and f(0) = 0, then f(x) is equal to (A) a + b (B) $\frac{ax^2}{2} + bx$ (C) $\frac{ax^2}{2} + bx + c$ (D) b	
Ans	**(B)** $\frac{ax^2}{2} + bx$	**1**
6.	Degree of the differential equation $\sin x + \cos\left(\frac{dy}{dx}\right) = y^2$ is (A) 2 (B) 1 (C) not defined (D) 0	
Ans	**(C) not defined**	**1**
7.	The integrating factor of the differential equation $(1 - y^2)\frac{dx}{dy} + yx = ay, (-1 < y < 1)$ is (A) $\frac{1}{y^2 - 1}$ (B) $\frac{1}{\sqrt{y^2 - 1}}$ (C) $\frac{1}{1 - y^2}$ (D) $\frac{1}{\sqrt{1 - y^2}}$	
Ans	**(D)** $\frac{1}{\sqrt{1-y^2}}$	**1**
8.	Unit vector along $\overrightarrow{PQ}$, where coordinates of P and Q respectively are (2, 1, –1) and (4, 4, –7), is (A) $2\hat{i} + 3\hat{j} - 6\hat{k}$ (B) $-2\hat{i} - 3\hat{j} + 6\hat{k}$ (C) $\frac{-2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$ (D) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$	
Ans	**(D)** $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$	**1**

9.	Position vector of the mid-point of line segment AB is $3\hat{i} + 2\hat{j} - 3\hat{k}$. If position vector of the point A is $2\hat{i} + 3\hat{j} - 4\hat{k}$, then position vector of the point B is (A) $\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2} - \frac{7\hat{k}}{2}$ (B) $4\hat{i} + \hat{j} - 2\hat{k}$ (C) $5\hat{i} + 5\hat{j} - 7\hat{k}$ (D) $\frac{\hat{i}}{2} - \frac{\hat{j}}{2} + \frac{\hat{k}}{2}$	
Ans	**(B)** $4\hat{i} + \hat{j} - 2\hat{k}$	**1**
10.	Projection of vector $2\hat{i} + 3\hat{j}$ on the vector $3\hat{i} - 2\hat{j}$ is (A) 0 (B) 12 (C) $\frac{12}{\sqrt{13}}$ (D) $\frac{-12}{\sqrt{13}}$	
Ans	**(A) 0**	**1**
11.	Equation of a line passing through point (1, 1, 1) and parallel to z-axis is (A) $\frac{x}{1} = \frac{y}{1} = \frac{z}{1}$ (B) $\frac{x-1}{1} = \frac{y-1}{1} = \frac{z-1}{1}$ (C) $\frac{x}{0} = \frac{y}{0} = \frac{z-1}{1}$ (D) $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$	
Ans	**(D)** $\frac{x-1}{0} = \frac{y-1}{0} = \frac{z-1}{1}$	**1**
12.	If the sum of numbers obtained on throwing a pair of dice is 9, then the probability that number obtained on one of the dice is 4, is : (A) $\frac{1}{9}$ (B) $\frac{4}{9}$ (C) $\frac{1}{18}$ (D) $\frac{1}{2}$	
Ans	**(D)** $\frac{1}{2}$	**1**
13.	Anti-derivative of $\frac{\tan x - 1}{\tan x + 1}$ with respect to x is (A) $\sec^2\left(\frac{\pi}{4} - x\right) + c$ (B) $-\sec^2\left(\frac{\pi}{4} - x\right) + c$ (C) $\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\| + c$ (D) $-\log\left\|\sec\left(\frac{\pi}{4} - x\right)\right\| + c$	

Ans	**(C)** $\log\left\|\sec\left(\frac{\pi}{4}-x\right)\right\|+c$	**1**
14.	If (a, b), (c, d) and (e, f) are the vertices of ΔABC and Δ denotes the area of ΔABC, then $\begin{vmatrix} a & c & e \\ b & d & f \\ 1 & 1 & 1 \end{vmatrix}^2$ is equal to (A) $2\Delta^2$ (B) $4\Delta^2$ (C) 2Δ (D) 4Δ	
Ans	**(B)** $4\Delta^2$	**1**
15.	The function $f(x) = x\|x\|$ is (A) continuous and differentiable at $x = 0$. (B) continuous but not differentiable at $x = 0$. (C) differentiable but not continuous at $x = 0$. (D) neither differentiable nor continuous at $x = 0$.	
Ans	**(A) continuous and differentiable at** $x = 0$	**1**
16.	If $\tan\left(\frac{x+y}{x-y}\right) = k$, then $\frac{dy}{dx}$ is equal to (A) $\frac{-y}{x}$ (B) $\frac{y}{x}$ (C) $\sec^2\left(\frac{y}{x}\right)$ (D) $-\sec^2\left(\frac{y}{x}\right)$	
Ans	**(B)** $\frac{y}{x}$	**1**
17.	The objective function $Z = ax + by$ of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true ? (A) $a = 9, b = 1$ (B) $a = 5, b = 2$ (C) $a = 3, b = 5$ (D) $a = 5, b = 3$	
Ans	**(C)** $a = 3, b = 5$	**1**
18.	The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and $\left(\frac{20}{3}, \frac{4}{3}\right)$. If $Z = 30x + 24y$ is the objective function, then (maximum value of Z – minimum value of Z) is equal to (A) 40 (B) 96 (C) 120 (D) 136	

Ans	**Give 1 Mark to those who have attempted as the correct option is not given**	**1**
	(Question Nos. 19 & 20 are Assertion-Reason based questions of 1 mark each)	
19.	**Assertion (A) :** Maximum value of $(\cos^{-1} x)^2$ is π^2. **Reason (R) :** Range of the principal value branch of $\cos^{-1}x$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.	
Ans	**(C) (A) is true but (R) is false**	**1**
20.	**Assertion (A) :** If a line makes angles α, β, γ with positive direction of the coordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. **Reason (R) :** The sum of squares of the direction cosines of a line is 1.	
Ans	**(A) Both (A) and (R) are true and (R) is the correct explanation of (A)**	**1**
	SECTION-B **(Question nos. 21 to 25 are very short Answer type questions carrying 2 marks each)**	
21.	(a) Evaluate $\sin^{-1}\left(\sin \frac{3\pi}{4}\right) + \cos^{-1}(\cos \pi) + \tan^{-1}(1)$. **OR** (b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.	
Ans	$\sin^{-1}\left(\sin\frac{3\pi}{4}\right)+\cos^{-1}(\cos\pi)+\tan^{-1}(1)= \frac{\pi}{4}+\pi+\frac{\pi}{4}$ $= \frac{3\pi}{2}$ **Or**  **Correct Graph** **Range:** $\left[\frac{\pi}{2}, \pi\right]$	**$1\frac{1}{2}$** **$\frac{1}{2}$** **1** **1**
22.	A particle moves along the curve $3y = ax^3 + 1$ such that at a point with x–coordinate 1, y–coordinate is changing twice as fast at x–coordinate. Find the value of a.	
Ans	**Differentiating equation $3y = ax^3 + 1$ with respect to 'x', $3\frac{dy}{dx} = 3ax^2$** **Taking $x = 1$, $\frac{dy}{dx} = 2$, $3(2) = 3a(1)^2 \Rightarrow a = 2$**	**1** **1**

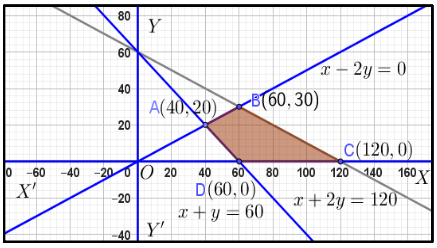
23.	If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero unequal vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}$, then find the angle between $\vec{a}$ and $\vec{b}-\vec{c}$.	
Ans	$\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}\Rightarrow\vec{a}\cdot(\vec{b}-\vec{c})=0\Rightarrow\vec{a}=\vec{0}$; $\vec{b}=\vec{c}$ or $\vec{a}\perp(\vec{b}-\vec{c})$ As, $\vec{a}\neq\vec{0}$; $\vec{b}\neq\vec{c}$ $\therefore$ the angle between $\vec{a}$ and $\vec{b}-\vec{c}$ is $\frac{\pi}{2}$	$1\frac{1}{2}$ $\frac{1}{2}$
24.	Find the coordinates of points on line $\frac{x}{1}=\frac{y-1}{2}=\frac{z+1}{2}$ which are at a distance of $\sqrt{11}$ units from origin.	
Ans	General point on the curve is $P(k, 2k+1, 2k-1)$, $k\in R$ $OP=\sqrt{11}\Rightarrow OP^2=11$ $\therefore k^2+(2k+1)^2+(2k-1)^2=11\Rightarrow k=\pm1$ $\therefore$ Coordinates of points are $(1,3,1)\,\&\,(-1,-1,-3)$	$\frac{1}{2}$ 1 $\frac{1}{2}$
25.	(a) If $y=\sqrt{ax+b}$, prove that $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2=0$. **OR** (b) If $f(x)=\begin{cases}ax+b & ; \ 0<x\le 1\\ 2x^2-x & ; \ 1<x<2\end{cases}$ is a differentiable function in (0, 2), then find the values of a and b.	
Ans	(a) $y=\sqrt{ax+b}\Rightarrow y^2=ax+b$ Differentiate with respect to 'x', $2y\frac{dy}{dx}=a$ Differentiate with respect to 'x', $2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=0\Rightarrow y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$ **Or** (b) $f(x)$ is differentiable in $(0,2)\Rightarrow f(x)$ is continuous on $(0,2)$ $\Rightarrow f(x)$ is continuous at $x=1$ $\therefore \lim_{x\to1^-}(ax+b)=\lim_{x\to1^+}(2x^2-x)\Rightarrow a+b=1$ Also, $f(x)$ is differentiable at $x=1$, $\therefore$ L.H.D.$(x=1)$ = R.H.D.$(x=1)$ $\Rightarrow a=4(1)-1\ \therefore a=3\ \&\ b=1-a=-2$	 1 1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$

SECTION-C (Question nos. 26 to 31 are short Answer type questions carrying 3 marks each)		
26.	(a) Evaluate $\int_0^{\pi/4} \log(1+\tan x)\,dx$. **OR** (b) Find $\int \frac{dx}{\sqrt{\sin^3 x \cos(x-\alpha)}}$.	
Ans	(a) Let $I=\int_0^{\pi/4}\log(1+\tan x)dx$ ---------- (i)	
	$\Rightarrow I=\int_0^{\pi/4}\log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx=\int_0^{\pi/4}\log\left(\frac{2}{1+\tan x}\right)dx$ ---------- (ii)	1
	Add (i) and (ii), $2I=\int_0^{\pi/4}\log 2\,dx=\log 2\cdot x\Big]_0^{\pi/4}=\frac{\pi}{4}\log 2$	1½
	$\Rightarrow I=\frac{\pi}{8}\log 2$	½
	Or	
	(b) Let $I=\int\frac{1}{\sqrt{\sin^3 x\cos(x-\alpha)}}dx=\int\frac{\operatorname{cosec}^2 x}{\sqrt{\sin\alpha+\cos\alpha\cot x}}dx$	1
	Put $\sin\alpha+\cos\alpha\cot x=t\Rightarrow \operatorname{cosec}^2 x dx=-\frac{1}{\cos\alpha}dt$	½
	$\therefore I=-\int\frac{1}{\cos\alpha\sqrt{t}}dt=-\frac{2\sqrt{t}}{\cos\alpha}+c$	1
	$\Rightarrow I=-\frac{2}{\cos\alpha}\sqrt{\sin\alpha+\cos\alpha\cot x}+c$	½
27.	Find $\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx$.	
Ans	Put $\cot^{-1}x=t\ \therefore x=\cot t$ and $\frac{1}{1+x^2}dx=-\,dt$	1
	$\therefore\int e^{\cot^{-1}x}\left(\frac{1-x+x^2}{1+x^2}\right)dx=-\int e^t\left(1-\cot t+\cot^2 t\right)dt=\int e^t\left(\cot t-\operatorname{cosec}^2 t\right)dt$	1
	$=e^t\cot t+c$	½
	$=xe^{\cot^{-1}x}+c$	½

28.	Evaluate $\displaystyle\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x + e^{-x})(e^x - e^{-x})}\,dx$	
Ans	**Let** $I = \displaystyle\int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{1}{(e^x+e^{-x})(e^x-e^{-x})}dx = \int_{\log\sqrt{2}}^{\log\sqrt{3}} \frac{e^{2x}}{(e^{2x})^2-1}dx$ **Put** $e^{2x} = t \Rightarrow e^{2x}dx = \frac{1}{2}dt$ **, Upper limit = 3, Lower Limit = 2** $\therefore I = \frac{1}{2}\displaystyle\int_2^3 \frac{1}{t^2-1}dt = \frac{1}{4}\log\left\|\frac{t-1}{t+1}\right\|\Big]_2^3$ $\Rightarrow I = \frac{1}{4}\left[\log\frac{2}{4} - \log\frac{1}{3}\right] = \frac{1}{4}\log\frac{3}{2}$	1 ½ 1 ½
29.	(a) Find the general solution of the differential equation : $(xy - x^2)\,dy = y^2\,dx.$ **OR** (b) Find the general solution of the differential equation : $(x^2+1)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$	
Ans	**(a) Given differential equation can be written as** $\frac{dy}{dx} = \frac{y^2}{xy-x^2} = \frac{\left(\frac{y}{x}\right)^2}{\frac{y}{x}-1}$ **Put** $y = ux, \frac{dy}{dx} = u + x\frac{du}{dx}$ $\therefore u + x\frac{du}{dx} = \frac{u^2}{u-1} \Rightarrow x\frac{du}{dx} = \frac{u}{u-1}$ **Separating the variables and integrating** $\displaystyle\int\left(1-\frac{1}{u}\right)du = \int\frac{dx}{x} \Rightarrow u - \log u = \log\|x\| + c \Rightarrow \frac{y}{x} - \log\frac{y}{x} = \log\|x\| + c$ **Or** **Rewriting the given differential equation as:** $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\sqrt{x^2+4}}{1+x^2}$ **Integrating factor** $= e^{\int\frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2$ $\therefore$ **solution of the differential equation is**	 ½ ½ ½ 1+½ ½ 1

	$y(1+x^2)=\int \frac{\sqrt{x^2+4}}{1+x^2}\cdot(1+x^2)dx+c$ $=\int \sqrt{x^2+4}dx+c$ $\therefore y(1+x^2)=\frac{x\sqrt{x^2+4}}{2}+2\log\left\|x+\sqrt{x^2+4}\right\|+c$	½ 1
30.	(a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable. **OR** (b) A and B throw a die alternately till one of them gets a '6' and wins the game. Find their respective probabilities of wining, if A starts the game first.	
Ans	**(a) X: Number of red balls out of the two balls drawn** X: 0, 1, 2 ; P(X): $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$ $\therefore \text{Mean}=0\cdot\frac{1}{4}+1\cdot\frac{1}{2}+2\cdot\frac{1}{4}=1$ **Or** **(b)** $P(\text{getting a six})=\frac{1}{6};P(\text{not getting a six})=\frac{5}{6}$ $P(\text{A wins})=\frac{1}{6}+\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+...=\frac{\frac{1}{6}}{1-\frac{25}{36}}=\frac{6}{11}$ $P(\text{B wins})=1-P(\text{A wins})=1-\frac{6}{11}=\frac{5}{11}$	½ 1½ 1 ½+½ 1½ ½
31.	Solve the following linear programming problem graphically : Minimize : $Z = 5x + 10y$ subject to constraints : $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0,\ y \geq 0$	

Ans



Correct graph — 2

Corner points	Value of Z
A (40,20)	400
B (60,30)	600
C (120,0)	600
D (60,0)	300 (Min)

Min(Z) = 300 at x = 60; y = 0 — 1

SECTION-D

(Question nos. 32 to 35 are Long Answer type questions carrying 5 marks each)

32. (a) If $A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations :

$x - 2y = 3$

$2x - y - z = 2$

$-2y + z = 3$

OR

(b) If $f(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, prove that $f(\alpha)\cdot f(-\beta) = f(\alpha - \beta)$

Ans

(a) $AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ — 1

$\Rightarrow B^{-1} = A$ — 1

The given system of equations can be written as:

$$B^T \cdot X = C, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, C = \begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$ — ½

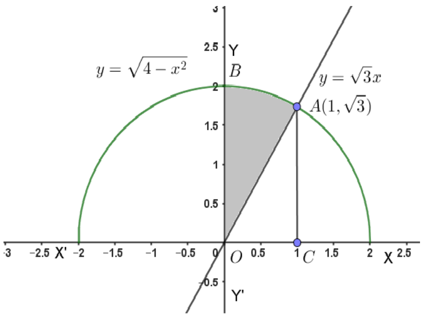
$$X = \left(B^T\right)^{-1} \cdot C = \left(B^{-1}\right)^T \cdot C = A^T \cdot C$$ — 1

$$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$$ — ½

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \therefore x = 1, y = -1, z = 1$$ — 1

Or

	(b) $LHS = f(\alpha)f(-\beta) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	1
	$= \begin{bmatrix} \cos\alpha\cos\beta + \sin\alpha\sin\beta & \cos\alpha\sin\beta - \sin\alpha\cos\beta & 0 \\ \sin\alpha\cos\beta - \cos\alpha\sin\beta & \sin\alpha\sin\beta + \cos\alpha\cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$	2
	$= \begin{bmatrix} \cos(\alpha-\beta) & -\sin(\alpha-\beta) & 0 \\ \sin(\alpha-\beta) & \cos(\alpha-\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$1\frac{1}{2}$
	$= f(\alpha-\beta) = RHS$	$\frac{1}{2}$
33.	(a) Find the equations of the diagonals of the parallelogram PQRS whose vertices are P(4, 2, –6), Q(5, –3, 1), R(12, 4, 5) and S(11, 9, –2). Use these equations to find the point of intersection of diagonals. **OR** (b) A line l passes through point (–1, 3, –2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.	
Ans	**Equation of diagonal PR:** $\frac{x-4}{8} = \frac{y-2}{2} = \frac{z+6}{11}$	1
	Equation of diagonal QS: $\frac{x-5}{6} = \frac{y+3}{12} = \frac{z-1}{-3}$	1
	General points on PR & QS are $(8k+4, 2k+2, 11k-6)$ **and** $(6t+5, 12t-3, -3t+1)$ **for real numbers 'k' and 't' respectively.**	1
	For point of intersection of PR and QS: $8k+4 = 6t+5,\ 2k+2 = 12t-3$	$\frac{1}{2}$
	Solving, we get $k = \frac{1}{2}, t = \frac{1}{2}$ $\therefore$ **The point of intersection is** $\left(8, 3, -\frac{1}{2}\right)$	$1\frac{1}{2}$
	Or	
	(b) Let direction ratios of the required line be a, b, c	
	Since it is perpendicular to the two given lines, $a+2b+3c=0$; $-3a+2b+5c=0$	1
	Solving together, $a=4k,\ b=-14k,\ c=8k$	1
	$\therefore$ **Equation of line is:** $\frac{x+1}{4k} = \frac{y-3}{-14k} = \frac{z+2}{8k} \Rightarrow \frac{x+1}{2} = \frac{y-3}{-7} = \frac{z+2}{4}$	1
	Vector equation: $\vec{r} = -\hat{i} + 3\hat{j} - 2\hat{k} + \lambda(2\hat{i} - 7\hat{j} + 4\hat{k})$	1

	Distance from origin $= \dfrac{\left\|\left(-\hat{i}+3\hat{j}-2\hat{k}\right)\times\left(2\hat{i}-7\hat{j}+4\hat{k}\right)\right\|}{\left\|2\hat{i}-7\hat{j}+4\hat{k}\right\|} = \dfrac{\left\|-2\hat{i}+\hat{k}\right\|}{\left\|2\hat{i}-7\hat{j}+4\hat{k}\right\|} = \dfrac{\sqrt{5}}{\sqrt{69}}$ **or** $\sqrt{\dfrac{5}{69}}$	1
34.	Using integration, find the area of region bounded by line $y=\sqrt{3}x$, the curve $y=\sqrt{4-x^2}$ and y–axis in first quadrant.	
Ans	 **Correct Figure** **Shaded region** **Point of intersection at** $x=1$ $ar(OAB)=\int_0^1\sqrt{4-x^2}\,dx-\int_0^1\sqrt{3}x\,dx$ $=\left(\dfrac{x\sqrt{4-x^2}}{2}+2\sin^{-1}\left(\dfrac{x}{2}\right)\right)\Big]_0^1-\dfrac{\sqrt{3}}{2}x^2\Big]_0^1$ $=\dfrac{\sqrt{3}}{2}+2\times\dfrac{\pi}{6}-\dfrac{\sqrt{3}}{2}=\dfrac{\pi}{3}$	1 ½ ½ 1½ 1 ½
35.	A function $f: [-4, 4] \rightarrow [0, 4]$ is given by $f(x)=\sqrt{16-x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a)=\sqrt{7}$.	
Ans	**Onto: Let** $y=\sqrt{16-x^2}\Rightarrow y\geq 0$ **Squaring we get,** $x^2=16-y^2\Rightarrow x=\pm\sqrt{16-y^2}$ **For each** $y\in[-4,4]$, **'x' is a real number,** $\therefore 0\leq y\leq 4\Rightarrow R_f=[0,4]=$ **Co – domain** $\therefore$ **'f' is an onto function.** **One-One:** $f(-1)=f(1)=\sqrt{15}$ **but** $-1\neq 1$, $\therefore$ **'f' is not a one-one function.** $f(a)=\sqrt{7}\Rightarrow\sqrt{16-a^2}=\sqrt{7}\Rightarrow a=\pm 3$	½ ½ 1½ 1½ 1

SECTION-E

(Question nos. 36 to 38 are source based/case based/passage based/integrated units of assessment questions carrying 4 marks each)

36. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine. The piston moves inside the cylinder bore

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area 75π cm^2.

Based on the above information, answer the following questions :

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

(ii) Find $\frac{dV}{dr}$.

(iii) (a) Find the radius of cylinder when its volume is maximum.

OR

(b) For maximum volume, h > r. State true or false and justify.

Ans

(i) $\pi r^2 + 2\pi rh = 75\pi \Rightarrow h = \frac{75 - r^2}{2r}$, $\therefore V = \pi r^2 h = \frac{\pi}{2}\left(75r - r^3\right)$ — **1**

(ii) $\frac{dV}{dr} = \frac{\pi}{2}\left(75 - 3r^2\right)$ — **1**

(iii) $\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0$ $\therefore$ volume is maximum when $r = 5$ — **2**

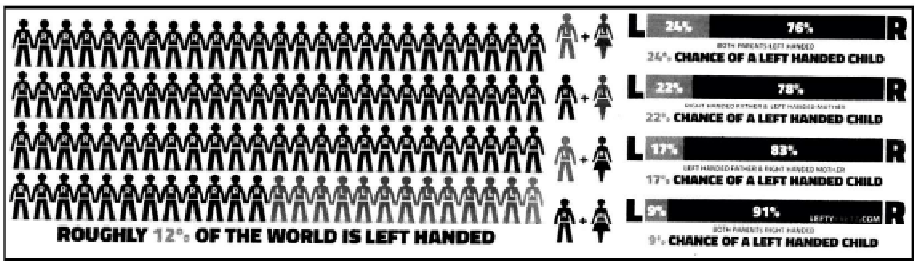
Or

False, — **½**

$\frac{dV}{dr} = 0 \Rightarrow r = 5, \left.\frac{d^2V}{dr^2}\right]_{r=5} = \frac{\pi}{2}(-6r) < 0$ $\therefore$ volume is maximum when $r = 5$ — **1**

As volume is maximum at $r = 5 \Rightarrow h = \frac{75 - 5^2}{2(5)} = 5 \Rightarrow h = r$ — **½**

37. Recent studies suggest that roughly 12% of the world population is left handed.



Depending upon the parents, the chances of having a left handed child are as follows :

A : When both father and mother are left handed :
Chances of left handed child is 24%.

B : When father is right handed and mother is left handed :
Chances of left handed child is 22%.

C : When father is left handed and mother is right handed :
Chances of left handed child is 17%.

D : When both father and mother are right handed :
Chances of left handed child is 9%.

Assuming that P(A) = P(B) = P(C) = P(D) = $\frac{1}{4}$ and L denotes the event that child is left handed.

Based on the above information, answer the following questions :

(i) Find P(L/C)

(ii) Find P($\bar{L}$/A)

(iii) (a) Find P(A/L)

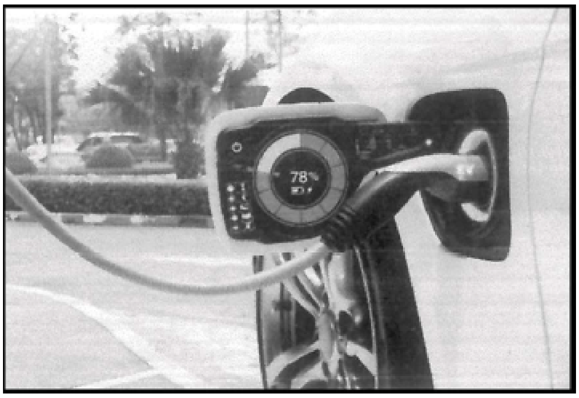
OR

(b) Find the probability that a randomly selected child is left handed given that exactly one of the parents is left handed.

Ans

(i) $P\left(L\,|\,C\right)=\frac{17}{100}$ **1**

(ii) $P\left(\bar{L}\,|\,A\right)=1-P\left(L\,|\,A\right)=1-\frac{24}{100}=\frac{76}{100}$ or $\frac{19}{25}$ **1**

	(iii) $P(A\mid L)=\dfrac{\frac{1}{4}\times\frac{24}{100}}{\frac{1}{4}\times\frac{24}{100}+\frac{1}{4}\times\frac{22}{100}+\frac{1}{4}\times\frac{17}{100}+\frac{1}{4}\times\frac{9}{100}}=\dfrac{24}{72}=\dfrac{1}{3}$ **Or** **Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.** $=P(L\mid B\cup C)=\dfrac{22}{100}+\dfrac{17}{100}=\dfrac{39}{100}$	2 2
38.	The use of electric vehicles will curb air pollution in the long run.  The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V : $V(t)=\frac{1}{5}t^3-\frac{5}{2}t^2+25t-2$ where t represents the time and t = 1, 2, 3.… corresponds to year 2001, 2002, 2003, ……. respectively. Based on the above information, answer the following questions : (i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify. (ii) Prove that the function V(t) is an increasing function.	

Ans	**(i) For the year 2000, $t = 0$ & $V(0) = -2$ and the number of vehicles cannot be negative** **$\therefore$ the given function V(t) cannot be used.**	2
	(ii) $V'(t) = \frac{3}{5}t^2 - 5t + 25 = \frac{3}{5}\left[\left(t - \frac{25}{6}\right)^2 + \frac{875}{36}\right] > 0$, $\therefore$ V(t) is an increasing function.	2