COMPETENCY–FOCUSED PRACTICE QUESTIONS

ISC–CLASS XII

MATHEMATICS

PREFACE

With a growing emphasis on competency-based education globally, the educational landscape in India has also steered towards high-quality learning experiences that allow learners to incorporate critical thinking and problem-solving approaches. This approach goes beyond rote memorisation and focuses on developing the skills and knowledge that students need to apply in their real-world scenarios.

The Council for the Indian School Certificate Examinations (CISCE), as a national-level progressive examination board, has taken several steps to infuse competency-based education in CISCE schools through teacher capacity-building on item development for competency-based assessments and the incorporation of competency-focused questions at the ICSE and ISC levels from the examination year 2024.

To further facilitate the adoption of competency-based assessment practices in schools and to support teachers and students towards the preparation for attempting higher-order thinking questions in future board examinations, Item Banks of **Competency-Focused Practice Questions** for selected subjects at the ICSE and ISC levels have been developed. This Item Bank consists of a rich variety of questions, both objective and subjective in categories, aimed at enhancing the subject-specific critical and analytical thinking skills of the students.

In this Item Bank, each question is accompanied by the topic and cognitive learning domain/s that it intends to capture. The cognitive domains reflected in these questions include understanding, analysis, application, evaluation and creativity, along with some questions of the higher-order recall domain. The Answer Key at the end presents the possible answers to a given question, but it is neither limiting nor exhaustive.

These practice questions are also meant to serve as teacher resources for classroom assignments and as samplers to develop their own repository of competency-focused questions. Apart from offering a good practice of higher order thinking skills, engaging with these questions would allow students to gauge their own subject competencies and use these *assessments for learning* to develop individual learning pathways.

During the development of this Item Bank, a large pool of questions was prepared by a team of experienced CISCE teachers. The questions that were finalised by the internal and external reviewers as being higher-order competency-focused questions have been collated in this item bank.

I acknowledge and appreciate all the ICSE and the ISC subject matter experts who have contributed to the development and review of these high-quality competency-focused questions for CISCE students.
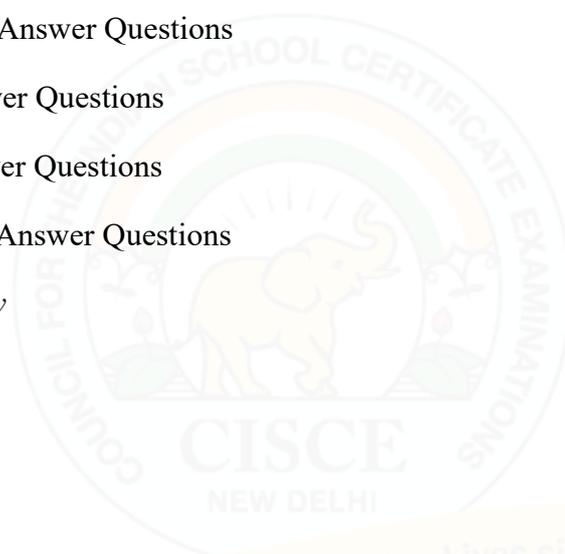
We are hopeful that teachers and students will utilise these questions to support their teaching-learning processes.

August 2024

Dr. Joseph Emmanuel
Chief Executive & Secretary
CISCE

Table of Contents

COMPETENCY-FOCUSED PRACTICE QUESTIONS

ISC-CLASS XII

Mathematics

I: Multiple Choice Questions	(1 Mark Each)

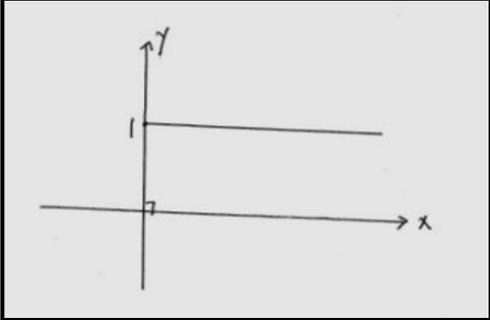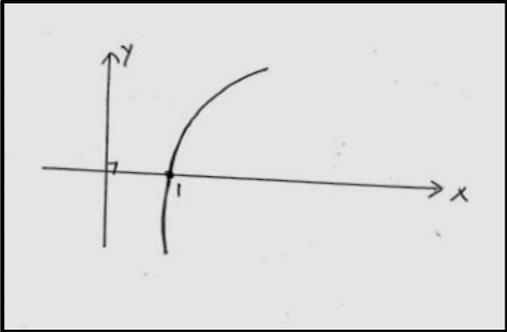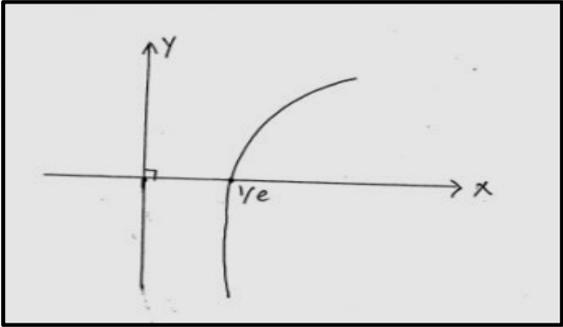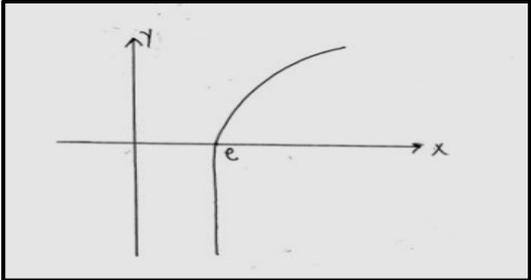Please select **ONE** correct answer from the following options provided in MCQs.

S.No.	Questions
1.	*[Relations and Functions]* If h(x)= 4^X and h^{-1} (x)=2, then value of x is: (a) -4 (b) 4 (c) -16 (d) 16 **(Application)**
2.	*[Inverse Trigonometric Functions]* If $a + \frac{\pi}{2} < 2\tan^{-1} x + 3\cot^{-1} x < b$, then a and b are respectively: (a) $\frac{\pi}{2}$ and 2π (b) $\frac{\pi}{2}$ and $-\frac{\pi}{2}$ (c) 0 and π (d) 0 and 2π **(Analysis)**
3.	*[Inverse Trigonometric Functions]* Which one of the following is *true*? (a) $\sin(\cos^{-1} x) = \cos(\sin^{-1} x)$ (b) $\sec(\tan^{-1} x) = \tan(\sec^{-1} x)$ (c) $\cos(\tan^{-1} x) = \tan(cos^{-1} x)$ (d) $\tan(\sin^{-1} x) = \sin(\tan^{-1} x)$ **(Application)**

S.No.	Questions
4.	*[Matrices]* If a matrix $A = [a_{ij}]_{2\times2}$, where , $a_{ij} = \begin{cases} 1 \, , & i \neq j \\ 0 \, , & i = j \end{cases}$, then A^{-1} is: (a) I (b) A (c) $-A$ (d) $-I$ **(Analysis)**
5.	*[Determinants]* If the value of 3rd order determinant is 5, then the value of determinant formed by replacing its element by its co-factor is: (a) 5 (b) $\frac{1}{5}$ (c) 125 (d) 25 **(Application)**
6.	*[Matrices]* If $A = \begin{bmatrix} 0 & 5 & -y \\ -5 & 0 & x \\ y & -x & 0 \end{bmatrix}$, then the value of $A^{-1}.(adj\ A)A$ is: (a) A^2 (b) I (c) 0 (d) A **(Analysis)**
7.	*[Determinants]* If $D = \begin{vmatrix} p & p & p \\ p & p+x & p \\ p & p & p+y \end{vmatrix}$ for $p \neq 0, x \neq 0, y \neq 0$ then D is divisible by: (a) only p. (b) p and x but not y. (c) p and y but not x. (d) p, x and y. **(Application)**

S.No.	Questions
8.	***[Determinants]***
	If adj(A) $= \begin{bmatrix} 2 & 3 & 5 \\ x & 5 & 1 \\ 3 & 3 & 4 \end{bmatrix}$ and $\lvert A \rvert = 4$, then the value of x is:
	(a) 16
	(b) 12
	(c) 32
	(d) 10 **(Application)**

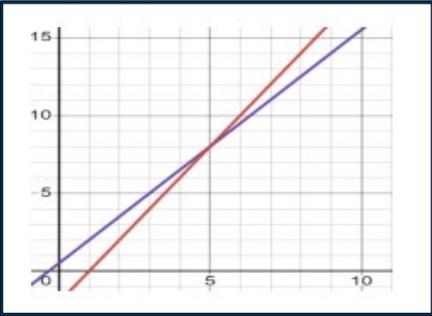| S.No.	Questions
9.	*[Differentiation]* Which of the following could be a sketch of the function $y = \dfrac{d}{dx}(x \log x)$? (a) (b) (c) (d) **(Analysis & Application)**

S.No.	Questions
10.	*[Integrals]* If, $\int \left(\frac{2-x}{(x-1)^2} \right) e^x dx = e^x f(x) + c$, then f(x) will be: (a) $\frac{1}{x-1}$ (b) $\frac{1}{2-x}$ (c) $\frac{1}{3-x}$ (d) $\frac{1}{1-x}$ **(Analysis & Application)**
11.	*[Differential Equations]* An integrating factor of the differential equation: $x \frac{dy}{dx} + y.P = x \cdot e^x \cdot x^{\frac{-1}{2} \log x}$, where P is a function of x, x > 0, is $\left(\sqrt{e} \right)^{(\log x)^2}$, then P is: (a) $(\log x)^2$ (b) $\frac{1}{\log x}$ (c) $\log x$ (d) $e^{\log x}$ **(Analysis & Evaluate)**
12.	*[Definite Integral]* Identify the *correct* answer of $\int_0^\pi Sin^{2024} x \, Cos^{2023} x \, dx$: (a) 0 (b) 1 (c) 2 (d) 3 **(Analysis)**

S.No.	Questions
13.	**[Probability]** Three fair dice are thrown. What is the probability of getting a total of 15 given that they exhibit three different numbers that are in arithmetic progression (A.P.)? (a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{8}$ (d) $\frac{1}{12}$ **(Application)**
14.	**[Probability]** Rohit and Vishal, two below-average students in a class, are attempting a Mathematics problem during revision classes. Their respective probabilities of solving the sum correctly are $\frac{1}{6}$ and $\frac{1}{8}$ respectively. Their previous experience shows that while solving the same question, the probability of a common mistake is $\frac{1}{10}$. What is the probability that they obtain the same answer? (a) $\frac{3}{4}$ (b) $\frac{7}{48}$ (c) $\frac{11}{96}$ (d) $\frac{9}{96}$ **(Application)**
15.	**[Vectors]** The value of $\hat{\imath} \cdot (\hat{k} \times \hat{\jmath}) + \hat{\jmath} \cdot (\hat{\imath} \times \hat{k}) + \hat{k} \cdot (\hat{\imath} \times \hat{\jmath})$ is: (a) -3 (b) -2 (c) -1 (d) 0 **(Application & Evaluate)**

S.No.	Questions
16.	**[Vectors]** Four students are playing a game. In a box, there are four strips of paper with four expressions written on each one. The student who picks up the meaningless expression will be out of the game. • Swati picks up the expression $\vec{a} \cdot (\vec{b} \times \vec{c})$. • Imran picks up the expression $\vec{a} \times (\vec{b} \times \vec{c})$. • Aryan picks up the expression $(\vec{a} \cdot \vec{b}) \times (\vec{c} \cdot \vec{d})$. • Maria picks up the expression $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d})$. **Who is out of the game?** (a) Swati (b) Imran (c) Aryan (d) Maria **(Understanding)**
17.	**[Three-Dimensional Geometry]** If L is the foot of the perpendicular drawn from a point P (a, b, c) on the XY plane, what is the co-ordinate of L? (a) (a, b, 0) (b) (0, 0, c) (c) (0, b, 0) (d) (a, 0, 0) **(Analysis & Application)**
18.	**[Three-Dimensional Geometry]** Find the equation of the plane passing through the point (2, -1,3) and containing the line: $$\frac{x-1}{3} = \frac{2-y}{4} = \frac{z+1}{2}$$ (a) $2x - 2y + z = 9$ (b) $2x + 2y + z = 5$ (c) $11x + 2y + z + 21 = 0$ (d) $11x - 2y + z + 17 = 0$ **(Application)**

S.No.	Questions
19.	*[Application of Calculus]* Rohit joins a career counselling institute as a counsellor. The manager says, "Within a year I want a breakeven point. For that, I will give you ₹ 24,000 fixed salary per month and the variable salary will be 25% of the revenue recovered on hiring students at the rate of ₹ 800/- charged from every student." Find how many students should be admitted by Rohit in a year in the institute to fulfil his manager's condition. (a) 30 (b) 40 (c) 80 (d) 100 **(Application)**
20.	*[Application of Calculus]* The fixed cost of a new product is ₹200 and the variable cost per unit is ₹ x. If the demand function p(x) = 30, then what are the value(s) of x that result in a loss given that x units of the product are sold? (a) $10 < x < 20$ (b) $x > 20$ (c) $x < 10$ (d) $x < 10$ or $x > 20$ **(Application)**
21.	*[Linear Regression]* In statistical modelling, regression analysis is a set of statistical processes for estimating the relationships between a dependent variable and one or more independent variables. The most common form of regression analysis is linear regression, in which one finds the line that most closely fits the data according to a specific mathematical criterion. *Source: https://en.wikipedia.org/wiki/Regression_analysis* If plotted on a graph, the independent variable is represented along _____ (a) Depends on the dataset. (b) Y axis. (c) X axis. (d) None of the above. **(Recall & Application)**

S.No.	Questions
22.	*[Continuity and Differentiability]* The function represented by the given graph is not differentiable at which of the following points: (a) -1, 0,1 (b) -1,1 (c) 0 (d) 1 **(Application)**
23.	*[Probability]* In an examination, a candidate takes three tests namely α, β, γ in succession and the probability of failing the first test α is $\frac{1}{2}$. The probability of passing each succeeding test is $\frac{1}{2}$ or $\frac{1}{4}$ according to whether he passes or fails in the preceding one. The candidate is selected, if he passes at least two tests. What is the probability that candidate is selected? (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{5}{8}$ (d) $\frac{3}{4}$ **(Application)**

S.No.	Questions
24.	*[Three-Dimensional Geometry]* In the picture given above, take a look at the double-arrowed lines drawn on the overpass. This is an example of skew lines in the real world. Based on this, which of these statements is **INCORRECT**? (a) These lines are not parallel. (b) These lines are intersecting. (c) These lines are not coplanar. (d) These lines can only exist in 3 or higher dimensional space. **(Application)**
25.	*[Linear Regression]* The graph below shows the two lines of regressions for a certain set of x and y values. What is the mean of x and mean of y? (a) 5 and 8 (b) 8 and 5 (c) (1, 0) (d) The mean of x and mean of y cannot be read from the graph. **(Analysis)**
26.	*[Application of Derivatives (Maximum & Minimum)]* A point $x = c$ is called the critical point of a function if: (a) $f'(c) = 0$ (b) f is not differentiable at $x = c$. (c) both (a) & (b). (d) none of the above. **(Recall & Understanding)**

II. Statement Based Questions   (1 Mark Each)

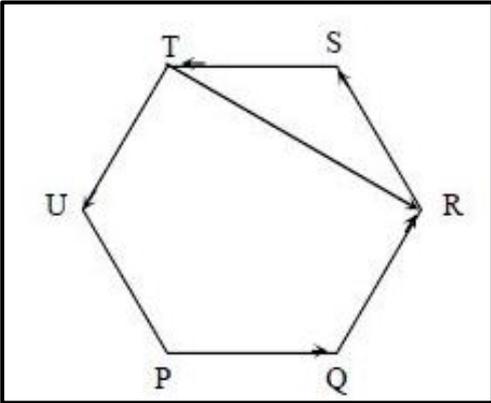The questions in this section have two/ more statements. Choose the correct option from the ones given below.

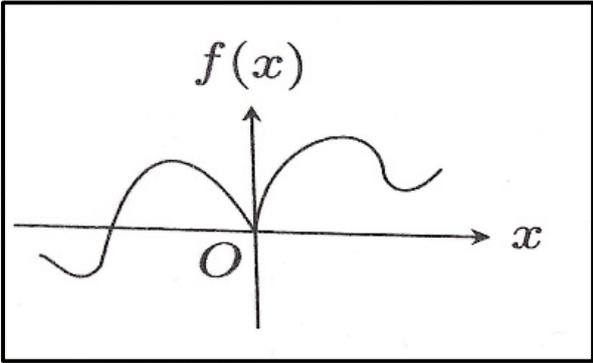| S.No.	Questions
27.	*[Differentiation]* Let $f(x)$ be a function such that $f'(x) = g(x)$ and $f''(x) = -f(x)$. Let $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. Then, consider the following statements: **Statement I:** $h'(2024) = 0$. **Statement II:** $h(2) = h\left(\frac{1}{2}\right)$ Which of the statements given above is/are *correct*? (a) I only. (b) II only (c) Both I and II (d) Neither I nor II. **(Analysis & Evaluate)**
28.	*[Relations and Functions]* **Statement I:** For any two real numbers a and b, we define a R b if $\quad sec^2 a - tan^2 b = 1$. Then, R is transitive. **Statement II:** The relation R on the set $\{2,3,4\}$ defined by R = $\{(2,2)\}$ is not symmetric. Which of the following options is *correct*? (a) Both the statements are true. (b) Both the statements are false. (c) Statement I is true, and Statement II is false. (d) Statement I is false, and Statement II is true. **(Analysis)**
29.	*[Relations and Functions]* **Statement I**: $f: R \to R$ given by $f(x) = \frac{1}{x^{-2}}$, is neither injective nor surjective. **Statement II:** $f: Z \to Z$ given by $f(x) = \sqrt[3]{x^9}$, is neither injective nor surjective. (a) Both the statements are true. (b) Both the statements are false. (c) Statement I is true, and Statement II is false. (d) Statement I is false, and Statement II is true. **(Analysis)**

S.No.	Questions
30.	**[Matrices and Determinants]** In the third-order matrix, a_{ij} denotes the element of the ith row and j^{th} column: $$a_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$$ **Statement I:** A Matrix is an upper triangular matrix. **Statement II:** The determinant of the matrix is equal to 1. Which of the above statement/s is/are *correct*? (a) Only I. (b) Only II. (c) Both I and II. (d) Neither I nor II. **(Analysis)**
31.	**[Application of Derivatives]** $y = x^5 + x^3, x \epsilon R$ is a function Rina, Abha and Saurabh have given their opinions about the function in the following statements: (i) Rina says that $'y'$ is an increasing function for all values of *x*. (ii) Abha says that $'y'$ is an odd function. (iii) Saurabh says that $'y'$ is symmetrical about the origin. Related to the above statements, which of the following option is *true*? (a) Rina, Abha and Saurabh are correct. (b) Rina and Abha are correct, but Saurabh is wrong. (c) Rina and Saurabh are correct, but Abha is wrong. (d) Saurabh and Abha are correct, but Rina is wrong. **(Analysis)**
32.	**[Calculus: Continuity]** **Statement I:** $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & if\ x \neq 0 \\ 0 & , if\ x = 0 \end{cases}$ is continuous at x= 0 but $f'(x)$ is not continuous at x=0. **Statement II:** The derivative of a continuous function need not be a continuous function. (a) Both (I) and (II) are correct and (II) is the correct explanation of (I). (b) Both (I) and (II) are correct and (II) is not the correct explanation of (I). (c) (I) is correct but (II) is incorrect. (d) (II) is correct but (I) is incorrect. **(Analysis)**

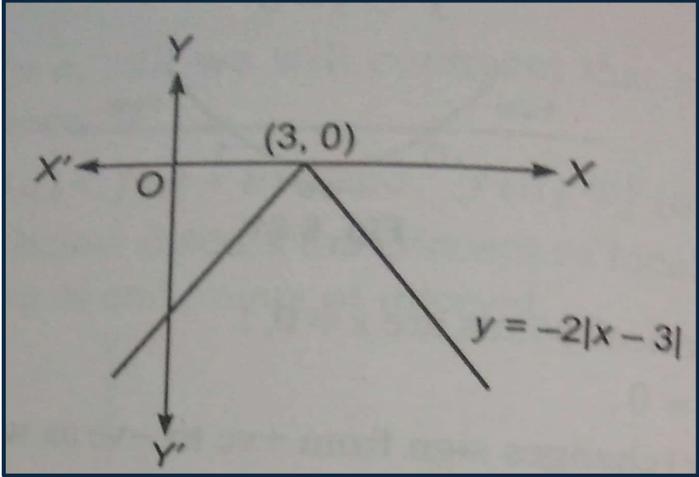| S.No.	Questions
33.	*[Definite Integral]* **Statement I:** $\int_{-2}^{2} \frac{x^2}{1 + 2^x} \, dx = \frac{8}{3}$. **Statement II:** $\int_{-a}^{a} f(x)dx = 2 \int_{0}^{a} f(x) \, dx$, if $f(x)$ is an even function. (a) Both the statements are true. (b) Both the statements are false. (c) Statement I is false, and statement II is true. (d) Statement I is true, and statement II is false. **(Understanding & Application)**
34.	*[Probability]* For any given events A: **Statement I**: Event A and null event $\emptyset$ are always independent. **Statement II**: Event A and sure event S are always independent. (a) Both the statements are true. (b) Both the statements are false. (c) Statement I is false, and statement II is true. (d) Statement I is true, and statement II is false. **(Analysis)**
35.	*[Linear Regression]* Which of the following statement(s) **DOES NOT/DO NOT** hold true related to regression analysis: **Statement I**: $|r|$ is the geometric mean of b_{yx} and b_{xy}. **Statement II**: b_{xy}, b_{yx} and r all are of the same sign. **Statement III**: The two regression lines do not intersect at $(\bar{x}, \bar{y})$. **Statement IV**: $-1 \le b_{yx} \times b_{xy} \le 1$. (a) III and IV only. (b) IV only. (c) III only. (d) II and III only. **(Recall & Analysis)**

S.No.	Questions
36.	**[Relations and Functions]**
	Assertion(A): The relation $f:\{m,n,p,q\}\rightarrow\{11,12,13,14\}$ defined by $f:\{(m,11),(n,12),(p,13)\}$ is a bijective function.
	Reason (R): The function $f:\{m,n,p\}\rightarrow\{11,12,13,14\}$ such that $f:\{(m,11),(n,12),(p,13)\}$ is one-one.
	(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A).
	(b) Both Assertion (A) and Reason (R) are true, and Reason (R) is not the correct explanation of Assertion (A).
	(c) Assertion (A) is true; Reason (R) is false.
	(d) Assertion (A) is false; Reason (R) is true. **(Understanding & Application)**
37.	**[Matrices]**
	Assertion (A): Let $A = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} d_1^{-1} & 0 & 0 \\ 0 & d_2^{-1} & 0 \\ 0 & 0 & d_3^{-1} \end{bmatrix}$
	Reason (R): If A is a diagonal matrix, then A^{-1} exists, it is also a diagonal matrix.
	(a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation for Assertion (A).
	(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation for Assertion (A).
	(c) Assertion (A) is true, and Reason (R) is false
	(d) Assertion (A) is false, and Reason (R) is true. **(Analysis)**

S.No.	Questions
38.	*[Probability]* If E_1 and E_2 are two mutually exclusive events associated with a random experiment and E is an event such that: $P(E) \neq 0$. **Assertion (A):** $P\left(\frac{E_1 \cup E_2}{E}\right) = P\left(\frac{E_1}{E}\right) + P\left(\frac{E_2}{E}\right)$. **Reason (R):** For two mutually exclusive events E_1 and E_2, $P(E_1 \cap E_2) = 0$. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation for Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (A) is not the correct explanation for Assertion (A). (c) Assertion (A) is true, and Reason (R) is false. (d) Assertion (A) is false, and Reason (R) is true.　　　　　**(Analysis)**
39.	*[Vectors]* The vectors $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU} \text{ and } \overrightarrow{UP}$ represent the sides of a regular hexagon. **Assertion(A):** $\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq \vec{0}$. **Reason(R):** $\overrightarrow{PQ} \times \overrightarrow{RS} = \vec{0}$ and $\overrightarrow{PQ} \times \overrightarrow{ST} \neq 0$. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of A. (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.　　　　**(Analysis)**

S.No.	Questions
40.	**[Application of Calculus]** **Assertion (A):** A company uses a demand function $p = \frac{a}{x+b} - c$, where $a, b, c \in R$ and $x = number$ of units. The Marginal Revenue decreases with the increase of x. **Reason (R):** $$\frac{d}{dx}(MR) < 0 \text{ , when } 0 < a < b.$$ Which one of the following options is *correct*? (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.　　　　　　　　　　　　**(Analysis)**
41.	**[Relations & Functions]** **Assertion (A):** The curve in the graph below is not a one-one function: **Reason (R):** If any straight line parallel to y-axis does not cut the curve at more than one point, then that curve represents a function. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true.　　　　　　**(Understanding)**

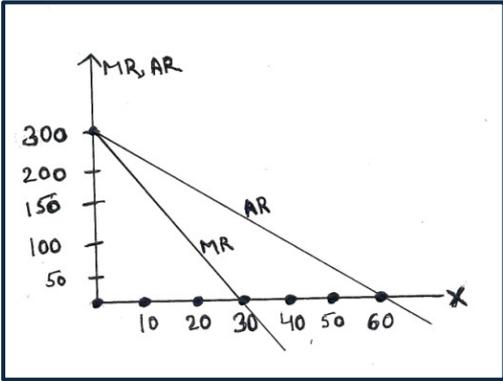| S.No.	Questions
42.	**[Application of Derivatives]** **Assertion (A) :** Let $f(x)$ be a polynomial function of degree 7 such that $\frac{d}{dx}\left(f(x)\right) = (x-2)^3(x+1)^2(7x-2)$ has a local minimum at $x = -1$. **Reason (R) :** Let f have first derivative at c such that $f'(c) = 0$ and $f'(c) > 0$, $\forall\, x \in$ $(c - \delta, c)$, $f'(c) < 0$, $\forall\, x \in (c,\, c + \delta,)$, then then c is a point of local minimum. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true. **(Application)**
43.	**[Differentiation]** **Assertion (A) :** If $y = \sin^{-1}\left(x\sqrt{x}\right)$, then $\frac{dy}{dx} = \frac{3\sqrt{x}}{2\sqrt{1-x^3}}$ **Reason (R) :** $\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$, $|x| \le 1$. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of A. (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true. **(Application)**
44.	**[Differential Equations]** **Assertion (A):** Degree of the differential equation $a\left(\frac{dy}{dx}\right)^2 + b\frac{dx}{dy} = c$ cannot be determined. **Reason (R):** If each term involving derivatives of a differential equation is a polynomial (or can be expressed as a polynomial) then the highest exponent of the highest order derivative is called the degree of the differential equation. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true. **(Recall & Analysis)**

S.No.	Questions
45.	**[Application of Derivatives]** Shown below the graph of $f(x) = -2\|x - 3\|$. **Assertion (A):** Maximum value of the function is 0. **Reason (R) :** Minimum value of the function approaches ∞. (a) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true, but Reason (R) is false. (d) Assertion (A) is false, but Reason (R) is true. **(Application)**

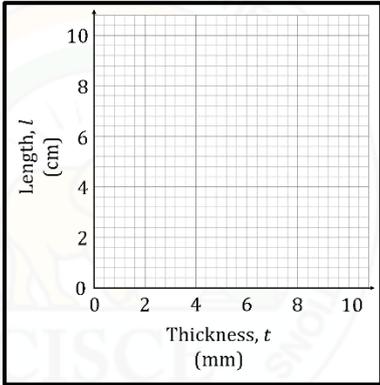## IV: Very Short Answer Questions   (1 Mark Each)

S.No.	Questions
46.	*[Inverse Trigonometric Functions]* If the domain of the function $f(x) = \sqrt{\cos^{-1}(3x) + \frac{\pi}{4}}$ is [a, b], then find the value of a+b. **(Analysis & Evaluate)**
47.	*[Determinants]* Each triangular face of the pyramid of Peace of Kazakhstan is made up of 25 smaller triangles as shown in figure below: Using the above information and concept of determinant answer the following question. If (1,2) and (3,6) is the co-ordinates of the two vertices of one of the smaller triangles and its area is 5 square cm, then find the equation of line on which the third vertex of the triangle lie. **(Analysis & Evaluate)**

S.No.	Questions
48.	**[Calculus: Differentiation]** The figure given below shows the graph of a function $y = f(x)$. What is the derivative of the function? **(Analysis & Application)**
49.	**[Three-dimensional Geometry]** Find the angle between the normal to the planes. $\vec{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 3$ and $\vec{r} \cdot (3\hat{\imath} + 2\hat{\jmath} - \hat{k}) + 5 = 0$. **(Application)**

S.No.	Questions
50.	*[Application of Calculus]* The Average Revenue (AR) and Marginal Revenue (MR) of a company on selling x units (on x-axis) is shown in the following graph: At how many units, should the manufacturer limit production so that each additional unit **DOES NOT** bring losses? **(Application)**
51.	*[Relations & Functions]* f(x) = x^2+2x-3 for x⩾-1. Given that the minimum value of x^2 +2x-3 occurs when x= -1, explain why f(x) has an inverse. **(Analysis)**
52.	*[Relations and Functions]* Explain why x + 4y = 12, x, y ∈ N is **NOT** a symmetric relation. **(Understanding & Application)**
53.	*[Inverse Trigonometric Function]* If $\cos^{-1}\frac{x}{a} - \cos^{-1}\frac{y}{b} = \frac{\pi}{3}$ and $\sin^{-1}\frac{x}{a} + \sin^{-1}\frac{y}{b} = \frac{2\pi}{3}$, then find the value of $4\frac{x^2}{a^2} + \frac{y^2}{b^2}$. **(Analysis & Application)**
54.	*[Matrices]* The matrix A = $\begin{bmatrix} a & 6 & b \\ 2c & x+1 & 8 \\ -1 & -8 & y-3 \end{bmatrix}$ is a skew symmetric matrix, then find the value of $ab + bc - xy$. **(Analysis & Application)**
55.	*[Application of Derivatives]* The displacement, x mt of a particle from a fixed point at time 't' seconds is given by $x = 6\cos(3t + \frac{\pi}{3})$. Find the acceleration of the particle, when $t = \frac{2\pi}{3}$. **(Application)**

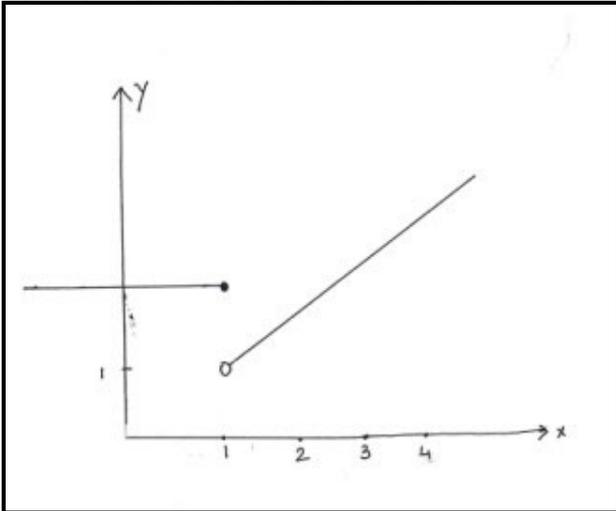| S.No.	Questions
56.	*[Definite Intervals]* Evaluate: $\int_{-\pi/2}^{\pi/2} Sin|x|dx$. **(Evaluate)**
57.	*[Probability]* Rahul, a class XII student, has a probability of getting a grade A in the examination of three subjects namely Mathematics, Physics and Chemistry are 0.2, 0.3 and 0.5 respectively. Find the probability that he gets a grade A in none of the subjects. **(Application)**
58.	*[Probability]* If A and B are two events such that $P(\bar{A}) = 0\cdot3, P(B) = 0\cdot4, P(A \cap \bar{B}) = 0\cdot5$, then find the value of $P(B / (A \cup \bar{B}))$. **(Analysis & Application)**
59.	*[Probability]* There are 10 cookies in a box. Six have chocolate centres and four have jam-filled centres. Shweta randomly chooses a cookie from the box and eats it. Then, Ali randomly chooses and eats one of the remaining cookies. What is the probability that Shweta and Ali choose cookies with different centres? **(Analysis & Application)**
60.	*[Probability]* A biased dice is tossed and the respective probabilities for various faces to turn up are: <table><tr><td>Face</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>Probability</td><td>.10</td><td>.24</td><td>.19</td><td>.18</td><td>.15</td><td>.14</td></tr></table> If an odd face has turned up, then what is the probability that it is face 1 or face 3? **(Analysis & Evaluate)**
61.	*[Vectors]* If $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then verify $\vec{a}, \vec{b}, \vec{c}$ are **NOT** mutually perpendicular. **(Application)**
62.	*[Vectors]* If $\vec{p} \times \vec{q} = \vec{0}$ and $\vec{p} \cdot \vec{q} = 0$, then what conclusion can we draw? **(Analysis)**
63.	*[Three-Dimensional Geometry]* A line makes angles α, β and γ with the x, y and z axes respectively. Given that $\alpha + \beta = \frac{\pi}{2}$, then what is the value of γ? **(Analysis & Application)**

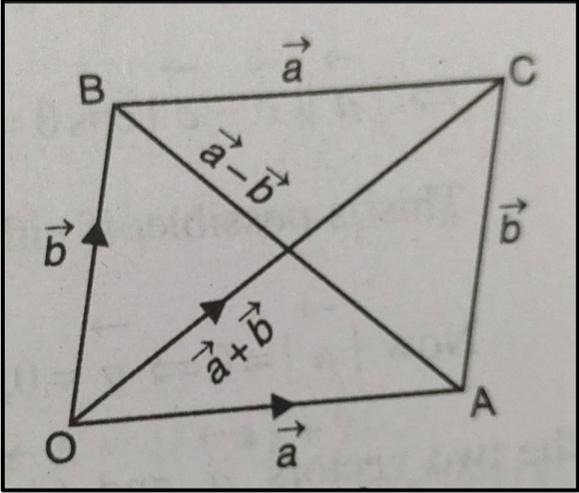| S.No.	Questions
64.	*[Three-Dimensional Geometry]* A variable plane at unit distance from the origin cuts the co-ordinate axes at A, B and C. The centroid of the triangle formed by joining the points A, B and C satisfies the relation $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = k$, then what is the value of k? **(Analysis & Application)**
65.	*[Three-Dimensional Geometry]* The angle between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane $2x - y + \sqrt{k}z + 4 = 0$ is α. Given $\sin \alpha = \frac{1}{3}$, what will be the value of k? **(Application)**
66.	*[Application of Calculus]* The cost function of a product is given by $C(x) = \frac{x^3}{3} - 45x^2 - 900x$ where x is the number of units produced. What will be the slope of marginal cost? **(Application)**
67.	*[Application of Calculus]* A company paid ₹ 8475 towards the rent of the building and ₹ 7625 interest on the loan. The cost of producing one unit of a product is ₹ 20. If each unit is sold for ₹ 27, find the break-even point. **(Application)**
68.	*[Application of Calculus]* The total cost of producing and marketing x units of bulbs by a whole seller is given by $C(x) = \frac{1}{3}x^2 + e^{2x} + 3e$ What will be the average cost of producing 3 units of bulbs? **(Application)**

S.No.	Questions
69.	*[Linear Regression]* Nikhil is investigating the growth patterns of a certain species of slug and measures their thickness, *t*, and length, *l*. The results are shown below: <table><tr><th>Thickness *t* (mm)</th><th>Length *l* (cm)</th></tr><tr><td>9</td><td>5.9</td></tr><tr><td>8</td><td>4.1</td></tr><tr><td>3</td><td>2.1</td></tr><tr><td>6</td><td>1.8</td></tr><tr><td>4</td><td>3.0</td></tr><tr><td>10</td><td>7.8</td></tr></table> Using scatter diagram, explain why a line of best fit should not be used for this data? **(Analysis)**
70.	*[Linear Regression]* If the covariance of x and y is 58.08, the variance of x is 121 and the standard deviation of y is 8 then, find the regression coefficient of x and y. **(Recall & Application)**

V. Short Answer Questions   (2 Marks Each)

S.No.	Questions
71.	*[Relations & Functions]* The values of two functions f and g for certain values of x are given in the following table: | x	-2	0	3
-----	-----	-----	-----
$f(x)$	-12	-4	8
$g(x)$	0	-12	30	 (a) Find the value of $f^{-1}(8)$? (b) Given that *f(x)* is a linear function, find *f(x)*? **(Analysis)**
72.	*[Relations & Functions]* "A function f is called a self-inverse function if $f^{-1}(x) = f(x)$ for all values of x in the domain." Let $f(x) = \dfrac{\pi^2}{x}$, where $x \neq 0, x \in R$. Show that $f(x)$ is a self-inverse function. **(Application)**
73.	*[Application of Derivatives]* Rahul sits on a speed boat in an island to do water sports, which is moving along a curve $y = \dfrac{1}{cosx\ sinx}$ in water. Find the number of stationary points. **(Analysis & Application)**
74.	*[Continuity & Differentiability]* The function defined by $f(x) = sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ is differentiable at $x = 0$. Is this statement *true or false*? Give reason for your answer. **(Application)**
75.	*[Application of Derivatives]* Seema enjoys a roller coaster ride in Ferrari world by first going downwards and then upwards to the maximum height. The relation between the distance travelled (cm) with respect to the time taken to complete the side by Seema is given by the following equation: $$y = 4x - \frac{1}{2}x^2$$ where $x = time\ in\ seconds$, (a) What is the rate of change of displacement with respect to the time? (b) How many seconds it will take her to go to its maximum height? **(Analysis & Evaluate)**

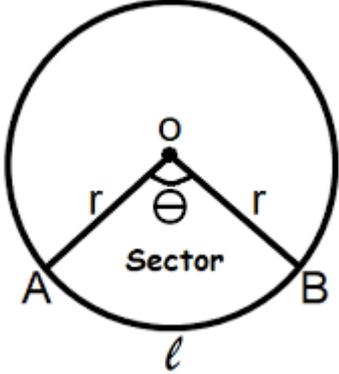| S.No.	Questions
76.	*[Differential Equations]* A child is solving an equation $x^2 + y^2 = 1$ and finds that this is the solution of the following differential equation: $1 + yy'' + (y')^2 = 0$. Justify his answer. **(Application)**
77.	*[Application of Derivatives]* Given $2f(x) = \log|x|^a - bx^2 + x$, if f(x) has extreme values at x = -1 and x = 2, find the value of a and b. **(Application)**
78.	*[Differential Equations]* The solution of $x\,dy - y\,dx = 0$ represents a family of straight lines passing through the origin. Justify. **(Application)**
79.	*[Differential Equations]* Find the function 'f' which satisfies the equation $\frac{df}{dx} = 2f$, given that $f(0) = e^3$. **(Analysis & Evaluate)**
80.	*[Integrals]* Evaluate: $\int 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx$. **(Application & Evaluate)**
81.	*[Integrals]* Evaluate: $\int \{f(ax+b)\}^n \cdot f'(ax+b)dx, n \neq -1$. **(Evaluate)**
82.	*[Integrals]* Evaluate: $\int \dfrac{dx}{x^{1/2} - x^{1/3}}$. **(Application & Evaluate)**
83.	*[Vectors]* Derive a condition for which given, $\vec{b} = \vec{c}$, we can say $\vec{a} \times \vec{b} = \vec{c} \times \vec{a}$? **(Analysis & Create)**
84.	*[Vectors]* If the angle between $\vec{a} = -3\hat{i} + x\hat{j} + \hat{k}$ and $\vec{b} = x\hat{i} + 2x\hat{j} + \hat{k}$ is acute and the angle between $\vec{b}$ and the x-axis lies between $\frac{\pi}{2}$ and π. Find the range of values for x. **(Analysis & Evaluate)**

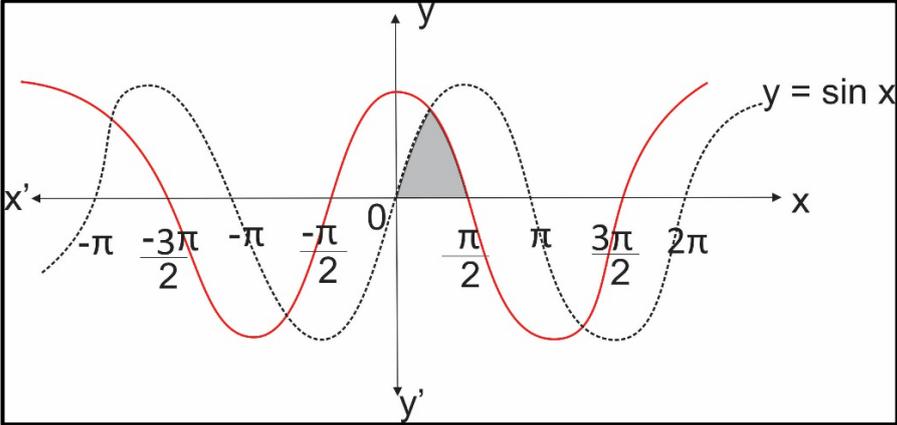| S.No.	Questions
85.	*[Application of Calculus]* To launch a new product in his company, Mr. Rajesh spends ₹ 1 lakh on the infrastructure and the variable cost of the product is estimated as ₹ 150/- per unit. The sale price per unit is fixed as ₹ 200/-. Additionally, Mr Rajesh also spends ₹ 0.5 per unit squared for marketing. Find the profit function. Hence, draw an inference regarding the breakeven point. **(Analysis & Evaluate)**
86.	*[Application of Calculus]* Anuj and Ashish launched a new fountain pen in their pen-factory which is consisting of ₹ 6400/- as overheads, ₹ 35/- per pen as the cost of material and labour cost, ₹ $\frac{x^2}{100}$ for x items produced. Find the values of x for which average cost is increasing. **(Analysis)**
87.	*[Inverse Trigonometric Functions]* If $\tan^{-1}\left(\frac{1}{1+1\cdot2}\right) + \tan^{-1}\left(\frac{1}{7}\right) + \cdots \cdot \tan^{-1}\left(\frac{1}{111}\right) = S$, then find tan S. **(Application & Evaluate)**
88.	*[Continuity & Differentiability]* The adjacent figure is the graph of function $y = f(x)$. Give answers to the following questions. (a) Name the type of discontinuity of the function. (b) Give reason for your answer. **(Analysis)**

S.No.	Questions
89.	**[Vectors]** If $\left\|\vec{a} - \vec{b}\right\| = \left\|\vec{a} + \vec{b}\right\|$ in the given figure, what inference can you draw? **(Analysis & Application)**
90.	**[Application of Calculus]** In Z Square Mall in Kanpur, there is a space to keep 300 cars and the entry fees per car is ₹ 20. It is estimated that if the entry fee is decreased by ₹ 5, then 50 additional cars can be adjusted in the same parking. Justify that the Marginal Revenue (MR) decreases at a higher rate than the Average Revenue (AR). **(Application)**
91.	**[Application of Differentiation (Tangent & Normal)]** Consider the functions $f(x) = -(x - h)^2 + 2k$. and $g(x) = e^{x-2} + k$, where $h, k \in$ R. (a) Find $f'(x)$. The graphs of f and g have a common tangent at x=3. (b) Show that: $2h = e + 6$. **(Understanding & Application)**
92.	**[Indefinite Integration]** Evaluate: $\int 2^x [f'(x) + f(x)\log 2]\, dx$. **(Understanding & Application)**

VI: Long Answer Questions (4 Marks Each)

S.No.	Questions
93.	**[Differentiation & Application of Differentiation]** Let $f(x) = \frac{\log 5x}{kx}$, where $x > 0$, $k \in R^+$. (a) Show that $f'(x) = \frac{1 - \log 5x}{kx^2}$. The graph of f has exactly one maximum point at P. (b) Find the x-coordinate of P. (c) Find $f''(x)$. (d) Find the value of x for which $f''(x)$ vanishes. **(Recall, Application & Evaluate)**
94.	**[Linear Programming]** A linear programming problem (LPP) is given as: Maximize $Z = x + 2y$ subject to the constraints $\quad x - y \geq 0, 2 \geq 2y - x, x \geq 0, y \geq 0$ Based on the above information, answer the following questions. (a) Find the corner points of the feasible region. (b) Find the corner point where maximum occurs. (c) Optimum solution does not exist. Justify your answer. **(Recall, Application & Evaluate)**
95.	**[Application of Integrals]** There are two curves given in the first quadrant as: $\quad x^2 + y^2 = \pi^2$, y = sinx. (a) What are the points of intersection of both the given curves? (b) What is the value of K, If the $\int_0^\pi \sqrt{\pi^2 - x^2}\ dx = \frac{\pi^3}{K}$? (c) Sketch the region enclosed by the given curves in the first quadrant and the y – axis. (d) Find the area of the region enclosed by the given curves in the first quadrant and the y – axis. **(Recall, Application & Evaluate)**

S.No.	Questions
96.	*[Vector Algebra]* Let $\vec{\alpha} = 3\hat{\imath} + \hat{\jmath}$ and $\vec{\beta} = 2\hat{\imath} - \hat{\jmath} + 3\ \hat{k}$. If $\vec{\beta} = \overrightarrow{\beta_1} - \overrightarrow{\beta_2}$ where $\overrightarrow{\beta_1}$ is parallel to $\vec{\alpha}$ and $\overrightarrow{\beta_2}$ is perpendicular to $\vec{\alpha}$. (a) Find $\overrightarrow{\beta_1}$. (b) Find $\overrightarrow{\beta_2}$. (c) Hence, find $\overrightarrow{\beta_1} \times \overrightarrow{\beta_2}$. **(Recall, Application & Evaluate)**
97.	*[Application of Differentiation Increasing and Decreasing Function]* Given $f(x) = 2\log(x - 2) - x^2 + 4x + 1$ and $f'(x) = \dfrac{-k(x-p)(x-q)}{(x-k)}$. (a) Find $k + p + q$. (b) The Statement "$f(x)$ is strictly increasing in $(-\infty, 1] \cup (2, 3]$" is false. Justify. (c) Hence, find the intervals(s) in which the function is strictly increasing. **(Understand, Application, Analysis & Evaluate)**

S.No.	Questions
98.	**[Application of Derivatives (Maximum & Minimum)]**

Case Study

The length of the perimeter of a slice of a pizza in the form of a sector of circle is 20 cm.

r be the radius of the circle, sectorial angle be θ radian and l be the length of the arc.

Based on the above information, answer the following questions.

  (a) Express the radius of the sector is expressed in terms of sectorial angle be θ radian.

  (b) Let A be the area of the slice. Then, express A in terms of r.

  (c) For the maximum value of A, find the value of the sectorial angle.

  (d) Maximum area of the slice of the pizza is _____.

(Recall, Application & Evaluate) |

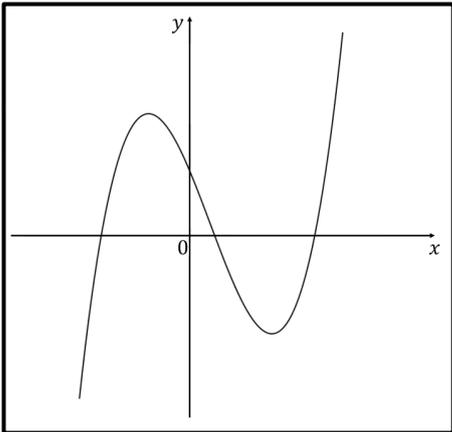| S.No.	Questions
99.	**[Application of Integration (Area under the curve)]** In a classroom, a teacher explains the properties of a particular curve by saying that this particular curve has beautiful ups and downs. It starts and heads down until π radian, and then heads up again and is closely related to sine function. Both follow each other at exactly $\frac{\pi}{2}$ radians apart as shown in the figure given below: Based on the above information, answer the questions that follow. (a) Name the curve that the teacher explained in the classroom. (b) Find the area of the curve explained in the passage from 0 to $\frac{\pi}{2}$ is. (c) Find the area of the shaded region. **(Application)**
100.	**[Relations & Functions]** (a) If a real-valued function is given by: $f(x) = \sqrt{25 - x^2}$ is an onto function, then find the co-domain for f(x). (b) If the domain is given to be [-5,5], is f(x) a one-one function? (c) Find all possible values of 'a' for which $f(a) = 4$. **(Analysis & Evaluate)**
101.	**[Inverse Trigonometric Functions]** Find the value of: $\cot \left[\sum_{n=1}^{25} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right]1.$ **(Analysis, Application & Evaluate)**
102.	**[Relations & Functions]** If $f : [1, \infty) \to [2, \infty)$ is given by $f(x) = x + \frac{1}{x}$ then, find $\frac{d}{dx} f^{-1}(x)$. **(Recall, Application & Evaluate)**

S.No.	Questions
103.	**[Application of Derivatives]** Find the acute angle between the curves $y = |x^2 - 1|$ *and* $y = |x^2 - 3|$ at their point of intersection when $x > 0$. **(Understanding, Application & Evaluate)**
104.	**[Application of Derivatives]** $y = \ln(x + 1) - \ln x$ is a curve. The tangent to the curve at the point P $(1, \ln 2)$ meets x-axis at A and y-axis at B. The normal to the curve at P meets the x-axis at C and y-axis at D. (a) Find the slope of tangent at P and find the slope of normal at P. (b) Find the equation of tangent at P. (c) Find the equation of normal at P. (d) Find the co-ordinates of A and C in terms of ln2. **(Understanding, Application & Evaluate)**
105.	**[Differential Equations]** Solve the following differential equation: $Cos^2 x \dfrac{dy}{dx} + y = tan\, x$, given that $y(0) = 1$. Hence, find $y\left(\dfrac{\pi}{4}\right)$. **(Analysis, Application & Evaluate)**
106.	**[Integrals]** Evaluate $\displaystyle\int \dfrac{dx}{\sqrt[4]{(x-1)^3 \cdot (x+2)^5}}$ **(Application, Analysis & Evaluate)**
107.	**[Definite Integrals]** For any function $f(x)$, we have $$\int_a^b f(x)\,dx = \int_a^{c_1} f(x)\,dx + \int_{c_1}^{c_2} f(x)\,dx + \ldots\ldots + \int_{c_n}^b f(x)\,dx.$$ Where $a < c_1 < c_2 < \ldots\ldots < c_n < b$. Based on above information, evaluate: $\displaystyle\int_0^2 |x^2 + 2x - 3|\,dx$. **(Create, Analysis, Application & Evaluate)**
108.	**[Probability]** A pot contains 5 red and 2 green balls. At random a ball is drawn from this pot. If a drawn ball is green, then put a red ball in the pot. If a drawn ball is red, then put a green ball in the pot. While drawn ball is not replaced in the pot. Now, we draw another ball randomly. What is the probability that the second ball drawn is a red ball? **(Application & Evaluate)**

S.No.	Questions
109.	**[Probability]** Rahul and Divya were playing the snakes and ladders board game. Each one had their own dice to play the game. Rahul was using a red dice, whereas Divya was using a black dice. In the beginning of the game, they were using their own dice to play. After some time, in order to play the game faster they both started using both the dice together for playing. When Divya rolled both red and black dice together then: (a) find the conditional probability of obtaining sum greater than 9, given that black dice resulted in a 5. (b) find the conditional probability that sum of the number on the dice is not 4, given that the numbers on the both the dice are different. **(Analysis & Application)**
110.	**[Three-dimensional Geometry]** Imagine you are at a point A, a café you visit often. Your friend is at the point B, a bookstore a few blocks away on a straight road. You want to meet your friend at a point on the line joining the café and the bookstore. Another friend, who is at home on the other side of the same road represented by point P, also wants to join. You decide to determine the exact meeting point by finding the foot of the perpendicular from P on the line joining the café and the bookstore. Given that co-ordinates of the café (A) are (1,2,4), of the bookstore(B) are (3,4,5) and the home are (2,1,3), find the location of the meeting point. **(Application & Evaluate)**
111.	**[Three-dimensional Geometry]** Two friends are planning a road trip. One friend stays in City A represented by the position vector $\left(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}\right)$ .The trip will start from City A and proceed towards the City B represented by the position vector $\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$. The friend living in City C represented by the position vector $7\hat{\imath} - \hat{k}$ will join when the first friend passes through her city. (a) Find the vector equation for the straight path between the cities A and B. (b) Hence, find out whether the three cities lie on the same straight path. (c) If the two friends now plan to travel $\sqrt{126}$ units along the vector $\overrightarrow{AB}$ from the City C, find the position vector of the destination point. **(Analysis, Application & Evaluate)**

S.No.	Questions
112.	**[Three-dimensional Geometry]** In the beautiful town of Darjeeling in the Himalayan foothills, the city planning committee wants to construct two major roads to connect the various neighbourhoods. The two roads are represented by the equations $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{k} = \frac{z-6}{7}$. As an in charge of the planning committee, ensure that these roads lie on the same plane to facilitate efficient urban planning and infrastructure development. (a) For what value of k, will the construction meet the requirement? (b) Hence, find the equation of the plane containing these lines. **(Recall, Analysis & Application)**
113.	**[Three-dimensional Geometry]** The plane px + y + pz = r intersects the co-ordinate axes at A, B and C. (a) Find the co-ordinates of the points A, B and C. (b) Find the co-ordinates of orthocentre of the triangle ABC. **(Recall, Understanding & Evaluate)**
114.	**[Application of Integrals]** From any point P (2, 1, 2) perpendiculars PM and PN are drawn to ZX and XY planes. (a) If O is the origin, find the equation of the plane OMN. (b) Find θ, if θ is the angle made by OP with the plane OMN. (c) If α, β and γ are the angles made by OP with the co-ordinate planes, prove that $\operatorname{cosec}^2 \theta = cosec^2 \alpha + cosec^2\beta + cosec^2 \gamma$. **(Analysis & Evaluate)**
115.	**[Application of Integrals]** Draw a rough sketch and find the area enclosed by the curve $y = -x^2$ and the line $x + y + 2 = 0$. **(Analysis & Evaluate)**
116.	**[Application of Integrals]** Find the area of the region bounded by the x-axis, part of the curve $y = 1 + \frac{8}{x^2}$, and the ordinates $x=2$ and $x=4$. If the ordinate at $x=a$ divides the area into two equal parts. Find the value of a. **(Analysis & Evaluate)**
117.	**[Application of Integrals]** Find the area enclosed between the co-ordinate axis and the curve $y^2 = 4a(x + \lambda)$ in the second quadrant. **(Analysis & Evaluate)**

S.No.	Questions
118.	**[Linear Regression]** A study was conducted to investigate the relationship between the number of hours a student studies per week (X) and their scores on a standardized test (Y). The following statistical data was collected from a sample of 50 students. <table><tr><td></td><td>X</td><td>Y</td></tr><tr><td>Mean</td><td>15</td><td>75</td></tr><tr><td>Standard Deviation (SD)</td><td>4</td><td>10</td></tr></table> The correlation coefficient between X and Y is 0.65. (a) Estimate the test score for a student who studies 20 hours per week. (b) If the pass mark is 40, then how many hours does a student need to study to pass? **(Application & Evaluate)**
119.	**[Linear Programming]** The corner points of the feasible region determined by the system of linear constraints are as shown below: Answer the following questions. (a) Let $Z = 3x - 4y$ be the objective function. Find the maximum and minimum value of Z and also the corresponding points at which the maximum and minimum value occurs. (b) Let $Z = px + qy$ where $p, q > 0$ be the objective function. Find the condition on p and q so that the maximum value of Z occurs at $B(4,10)$ and $C(6,8)$. (c) State the number of optimal solutions in this case. **(Application)**
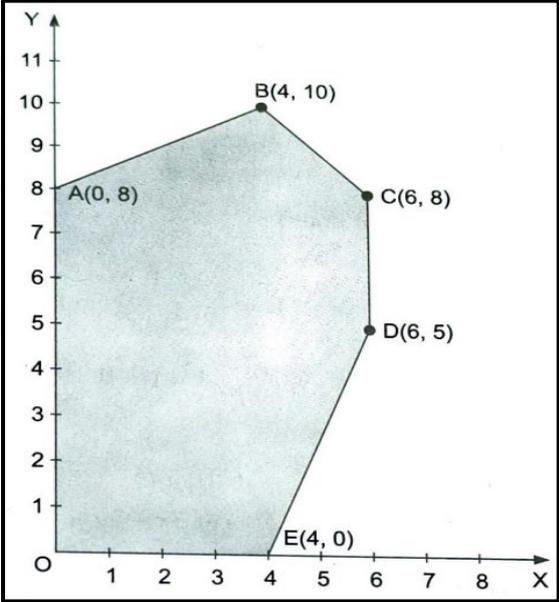
S.No.	Questions
120.	**[Linear Regression]** A movie cinema is considering significantly reducing the price of their popcorn as they believe their customers spend more on drinks when they buy popcorn. They recorded the following data of the daily revenue from popcorn, `x`, and the daily revenue from drinks, `y` over 8 randomly selected days. <table><tr><th>Popcorn revenue ('x')</th><th>Drinks revenue ('y')</th></tr><tr><td>14</td><td>22</td></tr><tr><td>12</td><td>23</td></tr><tr><td>12</td><td>17</td></tr><tr><td>14</td><td>24</td></tr><tr><td>16</td><td>18</td></tr><tr><td>10</td><td>25</td></tr><tr><td>13</td><td>23</td></tr><tr><td>12</td><td>24</td></tr></table> (a) Find $\bar{x}$, $\bar{y}$. (b) Using $\bar{x}$, $\bar{y}$, find regression coefficient of y on x. (c) The equation of the regression line y on x is in the form y = a + bx. Calculate the values of a and b. **(Application)**
121.	**[Relations & Functions]** A part of the graph of the function f (x) $=2x^3 - 3x^2 - 12x + 8$, x $\in \mathbb{R}$ is shown below: Answer the following questions. (a) Explain why 'f' does not have an inverse. (b) The domain of 'f' is now restricted to a $\leq$ x $\leq$ b where a < 0 and b > 0. a and b are chosen so that f has an inverse and the interval [a, b] is as large as possible. Find the domain and range of f $^{-1}$. **(Analysis & Evaluate)**

S.No.	Questions
122.	**[Probability]** In a Kabaddi league, two matches are being played between Jaipur and Delhi. It is assumed that the outcomes of two games are independent. The probability of Jaipur winning, drawing and losing the game against Delhi are $\frac{1}{2}$, $\frac{3}{10}$, and $\frac{1}{5}$ respectively. Each team gets 5 points for win, 3 points for draw and 0 point for loss in a game. After two games, find the probability that: (a) Jaipur has more points than Delhi. (b) Jaipur and Delhi have equal points. **(Application & Evaluate)**
123.	**[Three-dimensional Geometry]** Two drones are being used for soil analysis over an area of farmland. Drone **A** has been programmed to fly on the path given by $\vec{r} = 6\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ and drone **B** has been programmed to fly on the path $\vec{r} = -4\hat{\imath} - \hat{k} + \mu(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$. At what points on their respective paths should they reach, so that they will be closest to each other? **(Analysis & Evaluate)**

S.No.	*Questions*
124.	*[Linear Programming]* A manufacturing company produces two types of cell phones, Android and iOS. The company has resources to make at the most 300 sets a week. It takes ₹1800 to make an Android set and ₹ 2700 to make an iOS set. The company cannot spend more than ₹648000 a week to make cell phones. The company makes a profit of ₹ 510 per Android and ₹ 675 per iOS set. If x and y denote, respectively, the number of Android sets and iOS sets made each week, then formulate this problem as a Linear Programming Problem (LPP) given that the objective is to maximize the profit. Based on it, answer the questions that follow. (a) What will be the maximum profit function on x and y sets? *(write your objective based on the above data).* (b) What will be the values of your objective function in the feasible region? (*at corner points*) (c) At what point the maximum profit will occur? (d) What's the weekly cost (in ₹) of manufacturing the sets? **(Application)**

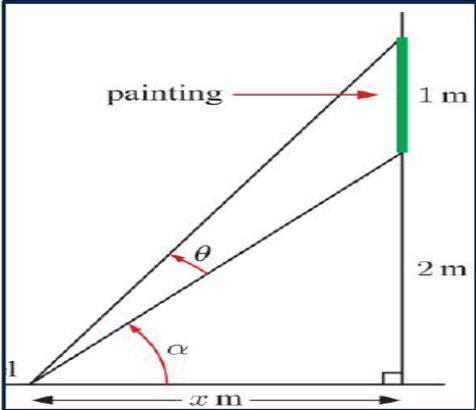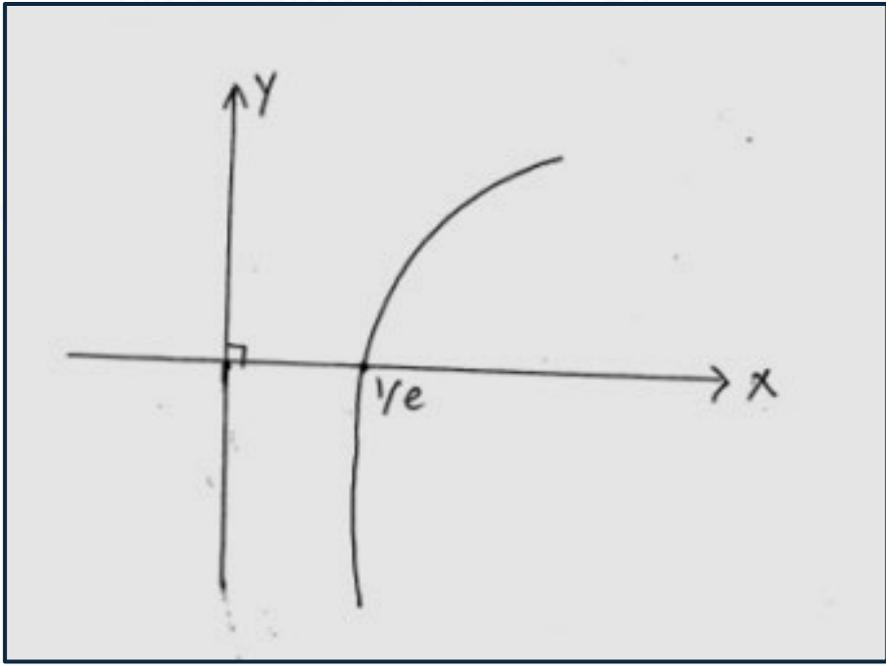## VII. Very Long Answer Questions   (6 Marks Each)

S.No.	Questions
125.	**[Determinants]** Rajesh wants to purchase some fruits from fruit market. 4 kilograms (kgs) apples, 3 kgs grapes and 2 kgs oranges cost him ₹600, 2 kgs apples, 4 kgs grapes and 6 kgs oranges cost him ₹900, and 6 kgs apples, 2 kgs grapes and 3 kgs oranges cost him ₹700. Using the given information, answer the following questions. (a) Express the given data in the form of a set of simultaneous equation. (b) Solve the set of simultaneous equations formed in sub part (a) by matrix method. (c) Hence, find how much Rajesh has to pay per kilogram (Kg) for each fruit. **(Create, Application & Evaluate)**
126.	**[Probability]** In a raffle draw, 1000 raffle tickets are sold for ₹1 each. Each has an equal chance of winning. First prize is ₹300, second prize is ₹200, and third prize is ₹100. Let X denote the net gain from the purchase of one ticket. (a) Construct the probability distribution of X. (b) Find the probability of winning any money in the purchase of one ticket. (c) Find the expected value of X and interpret its meaning. **(Create & Evaluate)**
127.	**[Probability]** Ram and Shyam play a game with a coin. Ram stakes ₹1.00 and throws the coin 4 times. If he throws 4 heads, he gets his stake and ₹3.00 from Shyam. If he throws only three heads and they are consecutive, he gets his stake and ₹2.00 from Shyam. If he throws only 2 heads and they are consecutive, he gets his stake and ₹1.00 from Shyam. In all other cases, Shyam takes the stake money. Is this game fair? Provide reasons for your answer. **(Analysis & Evaluate)**

S.No.	Questions
128.	**[Application of Calculus]** The fuel cost per hour for running a ship is proportional to the square of the speed generated in knots. The fuel cost is ₹ 75/h at 10 knots and the fixed charges amount to ₹ 1000/h. (a) Given that the fuel cost per hour is k times the square of the speed, the ship generates in km per hour, then what is the value of '**k**'? (b) What will be the cost per unit distance? (c) Determine the most economical speed to run the ship. **(Analysis & Evaluate)**
129.	**[Integrals]** Evaluate: $\int \dfrac{tan\frac{x}{4}dx}{1-sin\frac{x}{4}}$ **(Application & Evaluate)**
130.	**[Integrals]** Evaluate the following Integrals: $\int \left(\dfrac{cot\,x + cot^3\,x}{1+cot^3\,x}\right)dx$ **(Application & Evaluate)**
131.	**[Continuity & Differentiability]** Let $f(x) = \begin{cases} a + \sin^{-1}(x+b), x \geq 1 \\ x \qquad\qquad\quad ,x < 1 \end{cases}$, $f'(1)$ exists. (a) The statement "$f(x)$ is continuous at $x = 1$" is true. Justify. (b) Hence, find a relation between a and b. (c) Find $f'(x)$. (d) Hence, find the values of a and b. **(Recall, Understanding, Application & Evaluate)**

S.No.	Questions
132.	**[Differentiation & Application of Differentiation]** Let $f(x) = \cos x + \sqrt{3}\sin x$, $0 \le x \le 2\pi$. The following diagram shows the graph f. The $y-$intercept is at $(0, 1)$ and intersects $x-$axis at C and D. There is a *minimum point* at A(p, q) and a *maximum point* at B. Based on the above information, answer the questions that follow. (a) Write $f'(x)$ in the form of $\lambda\cos(x + \mu)$. (b) Find the value of q. (c) Find the coordinate of the point B. (d) Find the interval $f(x)$ is decreasing. (e) Find the slope of the tangent to the curve at D. (f) Find the slope of the normal to the curve at C. **(Understanding, Application, Evaluate & Create)**
133.	**[Differential Equations]** Consider the following differential Equation and answer the questions. $$\left[x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right] y\,dx - \left[y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right] x\,dy = 0$$ (a) Transform the above equation in the form $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$. (b) Use appropriate substitution to transform it into variable separable form. (c) Write the differential equation in variable separable form. (d) Prove that the solution of the differential equation is $\sec\left(\frac{y}{x}\right) = xyc$. (e) Find the solution if $x=1$, $y=1$. **(Analysis & Evaluate)**
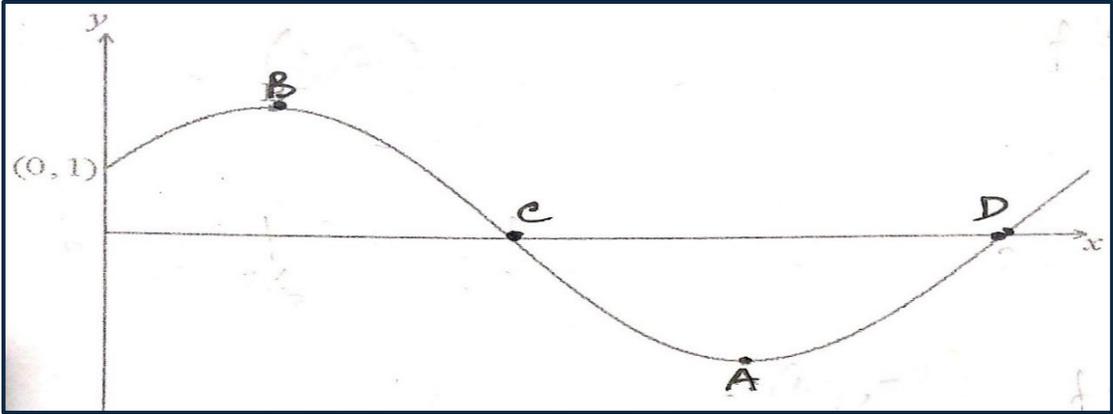
S.No.	Questions
134.	**[Application of Derivative (Maximum and Minimum)]** Sonia watches a painting which has its bottom edge 2 meters (m) above eye level and its top edge is 3 m above eye level as shown in the diagram. Based on the above information answer the questions that follow. (a) Given α and θ as shown in the diagram, find $\tan \alpha$ and $\tan(\alpha + \theta)$. (b) Find θ in terms of x only. (c) Find $\dfrac{d\theta}{dx}$. (d) Find x so that $\dfrac{d\theta}{dx} = 0$. (e) Use 1st derivative test, find the distance Sonia should stand from the wall to maximize her viewing angle of the painting. **(Recall, Application & Evaluate)**

S.No.	Expected Answers
1.	(d) 16
2.	(a) $\frac{\pi}{2}$ and 2π
3.	(a) $\sin(\cos^{-1} x) = \cos(\sin^{-1} x)$
4.	(b) A
5.	(d) 25
6.	(c) 0
7.	(d) p, x and y.
8.	(a) 16
9.	(c)
10.	(d) $\frac{1}{1-x}$
11.	(c) log x
12.	(a) 0

S.No.	Expected Answers
13.	(b) $\dfrac{1}{6}$
14.	(d) $\dfrac{9}{96}$
15.	(c) -1
16.	(c) Aryan
17.	(a) (a, b, 0)
18.	(b) $2x + 2y + z = 5$
19.	(b) 40
20.	(d) x < 10 or x > 20
21.	(c) X axis.
22.	(a) -1, 0,1
23.	(a) $\dfrac{3}{8}$
24.	(b) These lines are intersecting.
25.	(a) 5 and 8
26.	(c) both (a) & (b).
27.	(c) Both I and II.
28.	(c) Statement I is true, and Statement II is false.
29.	(c) Statement I is true, and Statement II is false.
30.	(b) Only II.
31.	(a) Rina, Abha, and Saurabh are correct.
32.	(a) Both (I) and (II) are correct and (II) is the correct explanation of (I).
33.	(a) Both the statements are true.
34.	(a) Both the statements are true.
35.	(a) III and IV only.

S.No.	Expected Answers
36.	(d) Assertion (A) is false, and Reason (R) is true.
37.	(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not correct explanation for Assertion (A).
38.	(a) Both Assertion(A) and Reason(R) are true, and Reason (R) is the correct explanation for Assertion (A).
39.	(c) Assertion(A) is true, but Reason (R) is false.
40.	(d) Assertion (A) is false, but Reason (R) is true.
41.	(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
42.	(d) Assertion (A) is false, but Reason (R) is true.
43.	(c) Assertion (A) is true, but Reason (R) is false.
44.	(d) Assertion (A) is false, but Reason (R) is true.
45.	(c) Assertion (A) is true, but Reason (R) is false.
46.	0
47.	Let the third vertex be (x, y), then we get $$\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = \pm 5$$ $6 - y - 6 + 2x + 3y - 6x = \pm 10$ $2y - 4x = \pm 10$ $y - 2x = \pm 5$
48.	$y = \lvert x + 3 \rvert$ $\dfrac{dy}{dx} = \dfrac{x + 3}{\lvert x + 3 \rvert}$
49.	90^0
50.	30 units. From the graph MR becomes zero at x=30. Hence, after 30 units MR becomes negative, so each additional unit thereafter bring losses.

S.No.	Expected Answers
51.	It is a one-one function because the function is always increasing for the given restricted domain. Reason-we are told that the function has a domain of x≥-1. So, it is only half of a quadratic graph. This means that we have a one-one function, since every point y will only have one value of x. It is also onto. Since one-one and onto functions are invertible, the given function is also invertible.
52.	x+ 4y = 12; x, y ∈ N The co-ordinate (4,2) satisfies the equation, x + 4y = 12. But the co-ordinates (2, 4) do not satisfy the equation x + 4y = 12. Hence, not a symmetric relation.
53.	2
54.	$ab + bc - xy$ $= 0 - 3 + 3 = 0$
55.	- 27 mt/sec2
56.	2
57.	0.280
58.	0.25
59.	$\dfrac{24}{45}$
60.	$\dfrac{29}{44}$
61.	$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ = **-7** i.e. ≠ 0. Hence $\vec{a}$, $\vec{b}, \vec{c}$ are not mutually perpendicular.
62.	Either $\vec{p} = \vec{0}$ or $\vec{q} = \vec{0}$
63.	$\dfrac{\pi}{2}$
64.	9
65.	$\dfrac{5}{3}$
66.	Slope of MC$= 2x - 90$.
67.	2300

S.No.	Expected Answers
68.	$AC = {} = 1 + \dfrac{e^6}{3} + e$
69.	 A line of best fit should not be used as the data is not linear.
70.	$r = \dfrac{58.08}{11 \times 8} = 0.66$ Regression coefficient of x and y (b_{xy}) $= 0.66 \times \dfrac{11}{8}$ $\therefore b_{xy} = 0.9075$
71.	$f(3) = 8$ $\therefore f^{-1}(8) = 3$ Since, f(x) is a linear function, So, the graph of y=f(x) is a straight line. $m = \dfrac{-4 + 12}{0 + 2} = \dfrac{8}{2} = 4$ $\therefore$ y = 4x + (-4) ($\because$ y – intercept = -4 from the table) y = 4x – 4 $\therefore$ f(x) = 4x - 4
72.	Let $y = f(x)$ $\therefore \; y = \dfrac{\pi^2}{x} \Rightarrow x = \dfrac{\pi^2}{y} \; f^{-1}(x) = \dfrac{\pi^2}{x}$ $\therefore f(x)$ is a self -inverse function.

S.No.	Expected Answers
73.	$y = \dfrac{1}{cosx\ sinx}$ $y = 2cosec2x$ $\dfrac{dy}{dx} = -\dfrac{4cos2x}{sin^2 2x}$ $\therefore 0 < x < \pi$ $\therefore$ It is not defined for $x = 0, \pi\ as\ sin2\pi = 0$ $\therefore$ for stationary points $\dfrac{dy}{dx} = 0$ $\Rightarrow cos2x = 0$ $\therefore 2x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$ $\therefore x = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$ 2 stationary points.
74.	The given function is not differentiable at $x = 0$. Hence the statement is false. $f(x) = Sin^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ $= \dfrac{\pi}{2} - cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$ $f(x) = \begin{cases} \dfrac{\pi}{2} - 2tan^{-1}x, x \geq 0 \\ \dfrac{\pi}{2} + 2tan^{-1}x, x < 0 \end{cases}$ $f'(x) =$ $\begin{cases} \dfrac{-2}{1+x^2}, x > 0 \\ not\ derivable, x = 0 \\ \dfrac{2}{1+x^2}, x < 0 \end{cases}$

S.No.	Expected Answers
75.	$\dfrac{dy}{dx} = 4 - \dfrac{1}{2} \times 2x$ $\qquad = 4 - x.$ To reach maximum height $\qquad \dfrac{dy}{dx} = 0$ $\qquad \therefore 4 - x = 0$ $\qquad \therefore x = 4\ sec$ and $\qquad \dfrac{d^2y}{dx^2} = -1 < 0$ $\therefore$ Height is maximum after 4 seconds.
76.	$x^2 + y^2 = 1$ $2x + 2yy' = 0$ $x + yy' = 0$ Again differentiating $1 + yy'' + (y')^2 = 0$
77.	$f(x) = \dfrac{a}{2}\log|x| - \dfrac{1}{2}bx^2 + \dfrac{1}{2}x$ $f'(x) = \dfrac{a}{2x} - bx + \dfrac{1}{2}$ At x = -1, -a + 2b = -1 At x = 2, a − 8b = -2 a = 2, b = $\dfrac{1}{2}$
78.	$x\,dy - y\,dx = 0$ $\Rightarrow \dfrac{dy}{y} = \dfrac{dx}{x}$ $\log y = \log x + \log c$ $\Rightarrow y = cx$ which represents a family of straight lines passing through the origin.

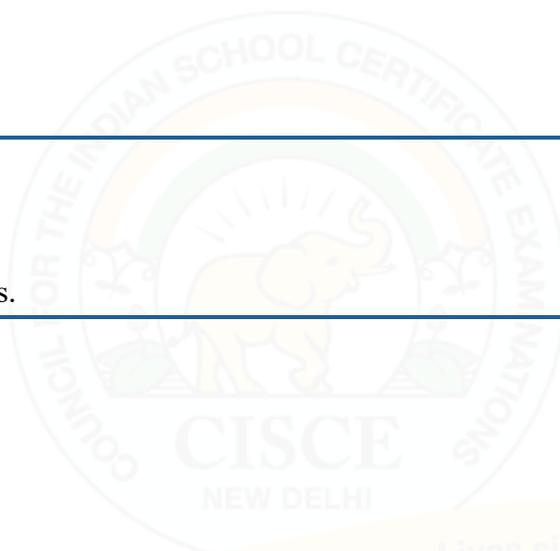| S.No.	Expected Answers
79.	$\dfrac{df}{f} = 2dx$ $\log f = 2x + \log c$ $\log\left(\dfrac{f}{c}\right) = 2x$ $f(0) = e^3 => f = e^3$ when $x = 0$ $e^3 = c.e^0$ $\qquad c = e^3$ $\therefore \qquad f = e^{2x+3}$
80.	Let $\dfrac{d}{dx} 2^{2^{2^x}} = t$ $\Rightarrow \quad 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x (\log 2)^3 = \dfrac{dt}{dx}$ $\Rightarrow \quad 2^{2^{2^x}} \cdot 2^{2^x} \cdot 2^x dx = \dfrac{dt}{(\log 2)^3}$ $\therefore \displaystyle\int \dfrac{dt}{(\log 2)^3}$ $= \dfrac{2^{2^{2^x}}}{(\log 2)^3} + c$
81.	Let, $f(ax + b) = t \Rightarrow f'(ax + b).adx = dt$ $I = \int t^n \cdot \dfrac{dt}{a}$ $= \dfrac{1}{a(n + 1)} \left(f(ax + b)\right)^{n+1} + c, \qquad n \neq -1$
82.	let $x = t^6$ $6t^5 = dx$ $6\displaystyle\int \dfrac{t^5}{t^3 - t^2} dt$ $= 6\displaystyle\int \left(t^2 + t + 1 + \dfrac{1}{t-1}\right) dt$ $6\left[\dfrac{t^3}{3} + \dfrac{t^2}{2} + t + \log(t - 1)\right] + c$ $= 6\left[\dfrac{1}{3}\sqrt{x} + \dfrac{1}{2}\sqrt[3]{x} + x^{\frac{1}{6}} + \log\left(x^{\frac{1}{6}} - 1\right)\right] + c$

S.No.	Expected Answers
83.	$\vec{a} \times \vec{b} = \vec{c} \times \vec{a}$ $\Rightarrow \vec{a} \times \vec{b} - \vec{c} \times \vec{a} = \vec{0}$ $\Rightarrow \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$ $\Rightarrow \vec{a} \times (\vec{b} + \vec{c}) = \vec{0}$ $\Rightarrow \vec{a} \times (\vec{b} + \vec{b}) = \vec{0}$ $\Rightarrow \vec{a} \times 2\vec{b} = \vec{0}$ $\Rightarrow 2(\vec{a} \times \vec{b}) = \vec{0}$ Condition will be $\vec{a} \times \vec{b} = \vec{0}$.
84.	$\vec{a} \cdot \vec{b} > 0$ as the angle is acute. $\Rightarrow -3x + 2x^2 + 1 > 0$ $\Rightarrow (x-1)(2x-1) > 0$ $\Rightarrow x < \frac{1}{2}$ or $x > 1$ $\vec{b} \cdot \hat{\imath} < 0$ as the angle lies between $\frac{\pi}{2}$ and π $\Rightarrow x < 0$
85.	$C(x) = 1,00,000 + 150x + 0.5x^2$ $R(x) = 200x$ Profit function $= P(x)$ $= R(x) - C(x)$ $= 200x - 1,00,000 - 150x - 0.5x^2$ For breakeven point, $P(x) = 0$ $\Rightarrow 0.5x^2 - 50x + 1,00,000 = 0$ $\Rightarrow x^2 - 100x + 2,00,000 = 0$ The discriminant is negative for the above quadratic equation. So, it is not possible to reach the breakeven point.

S.No.	Expected Answers
86.	The cost of material for producing x pens = ₹ 35 x overheads cost ₹ 6400/- and labour cost of producing x items = ₹ $\frac{x^2}{100}$ ∴ Total cost of producing x items. $C(x) = Rs.\left(6400 + 35x + \frac{x^2}{100}\right)$ ∴ Average cost $AC = \frac{C(x)}{x}$ $= ₹ \left(\frac{6400}{x} + 35 + \frac{x}{100}\right) \frac{d}{dx}\left(\frac{6400}{x} + 35 + \frac{x}{100}\right)$ $= \frac{-6400}{x^2} + \frac{1}{100}$ $= -\frac{(x+800)(x-800)}{100x^2} > 0$ for $-800 < x < 800$ But x can't be negative. Hence, $0 < x < 800$.
87.	$\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \cdots \tan^{-1}\left(\frac{1}{1+10.11}\right) = S$ $\therefore \tan^{-1}\left(\frac{2-1}{1+2.1}\right) + \tan^{-1}\left(\frac{3-2}{1+3.2}\right) + \cdots \tan^{-1}\left(\frac{11-10}{1+11\cdot10}\right) = S$ $(\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + \cdots.... +(\tan^{-1} 11 - \tan^{-1} 10) = S$ $\tan^{-1} 11 - \tan^{-1}1 = S$ $\tan^{-1}\frac{10}{12} = S$ $\frac{5}{6} = \tan S$
88.	(a) Non removable discontinuity. (b) From the graph,at x =1, LHL ≠ RHL. At x=1, we cannot infer what the limit is, as L.H.L.=2 and R.H.L.=1. So the function is not continuous as there is jump discontinuity. Hence, the function has non removable discontinuity at x =1.
89.	$\left|\vec{a} - \vec{b}\right| = \left|\vec{a} + \vec{b}\right|$ $\Rightarrow \left|\vec{a} + \vec{b}\right|^2 = \left|\vec{a} - \vec{b}\right|^2$ $\Rightarrow (a + b)^2 = (a - b)^2$ $\vec{a}^2 + 2\vec{a} \cdot \vec{b} + \vec{b}^2 = \vec{a}^2 - 2\vec{a} \cdot \vec{b} + \vec{b}^2$ $\Rightarrow 4\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0$ $\Rightarrow a \perp b$ The diagonals of a parallelogram are equal if it is a rectangle.

S.No.	Expected Answers
90.	Let ₹ x per car be the parking fee and y be the no. of cars standing in the parking area ∴ Revenue $= Rs.xy$ Let $y = a + bx$, where a and b are constants It is given *that* $y = 300$ *when* $x = 20$ *and* $y = 350$ *when* $x = 15$ ∴ $300 = a + 20b$ (i) and $350 = a + 15b$ (ii) Solving simultaneously, we get $a = 500, b = -10$ $y = a + bx$ $y = 500 - 10x$ ∴ R(Revenue) $= Rs\ xy$ $= x(500 - 10x)$ $R(x) = 500x - 10x^2$ ⇒MR $= 500 - 2x$ ⇒MR $= 0$ when x $= 25$ Also, AR $= 500 - 10x$ AR $= 0$ when x $= 50$. ∴ MR decreases at a higher rate than AR.
91.	(a) $f'(x) = -2(x - h)$ (b)∵, *f (x)* and *g(x)* have common tangent at x=3. ⇒slopes of the tangents to the two curves at x=3 are equal. ⇒$f'(x) = g'(3)$ [∵g(x)=$e^{x-2} + k$ ⇒-2 (3-h)= e g'(x)= e^{x-2} ∴ 2h= e +6 ⇒g' (3)= e] Hence, proved.
92.	$2^x f(x) + c$

S.No.	Expected Answers
93.	(a) $f(x) = \dfrac{\log 5x}{Kx}$, $x > 0$, $k \in R^+$. $= \dfrac{1}{K} \cdot \log \dfrac{5x}{x}$ Therefore, $f'(x) = \dfrac{1}{K} \dfrac{x \cdot \frac{d}{dx}(\log 5x) - \log 5x \cdot \frac{d}{dx}(x)}{x^2}$ $= \dfrac{1}{K} \cdot \dfrac{x \cdot \frac{1}{5x} \cdot 5 - \log 5x}{x^2}$ $f'(x) = \dfrac{1 - \log 5x}{kx^2}$ Hence, proved. (b) x −coordinate of P is $\dfrac{e}{5}$ (c) $f''(x) = \dfrac{(2\log 5x - 3)}{kx^3}$ (d) $\dfrac{1}{5} e^{\frac{3}{2}}$
94.	(a) (0,0), (2,2). (b) (2,2) (c) At (2,2), z = x +2y = 6. As the feasible region is unbounded, hence, largest value 6 may or may not be maximum. After plotting half plane x +2y > 6, we found that there are common points with feasible region. Hence, optimum solution does not exist.
95.	(a) (0,0) and (0, π) (b) k=4 (c) (d) $= \dfrac{\pi^3}{4} - 2s$

S.No.	Expected Answers
96.	(a) $\overrightarrow{\beta_1} = \frac{3}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$
	(b) $\overrightarrow{\beta_2} = -\frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} - 3\hat{k}$
	(c) $\overrightarrow{\beta_1} \times \overrightarrow{\beta_2} = -\frac{3}{2}\hat{\imath} + \frac{9}{2}\hat{\jmath} + \frac{5}{2}\hat{k}$
97.	(a) $k + p + q = 6$.
	(b) $f'(x) \geq 0$, $\forall x \in (-\infty, 1] \cup (2,3]$. But domain of $f(x)$ is $\in (2, \infty)$.
	$\therefore$ The Statement "$f(x)$ is strictly increasing in $(-\infty, 1] \cup (2,3]$" is false.
	(c) $f(x)$ is strictly increasing in $(2,3]$.
98.	(a) $r = \frac{20}{\theta + 2}$
	(b) $10r - r^2$
	(c) 2^c
	(d) $25\ cm^2$
99.	(a) cosine.
	(b) 1 sq unit.
	(c) $2 - \sqrt{2}$ sq. units.

S.No.	Expected Answers
100.	(a) $y = \sqrt{25 - x^2}$ $\Rightarrow x = \sqrt{25 - y^2}$ $25 - y^2 \geq 0$ $\Rightarrow (5 - y)(5 + y) \geq 0$ $\Rightarrow -5 \leq y \leq 5$ But, $y \geq 0$ $\therefore$ Range is [0.5]. Given, f(x) is an onto function the range is equal to the co-domain. $\therefore$ co-domain of f(x) is [0,5] (b) Let $x_1, x_2 \in [-5,5]$. If $f(x_1) = f(x_2)$ $\Rightarrow \sqrt{25 - x_1^2} = \sqrt{25 - x_2^2}$ $\Rightarrow x_1 = \pm x_2$ $\therefore$ f(x) is not one-one in the given domain. (c) f(a)=4 $\Rightarrow \sqrt{25 - a^2} = 4$ $\Rightarrow 25 - a^2 = 16$ $\Rightarrow a^2 = 9$ $\therefore a = \{-3,3\}$

S.No.	Expected Answers
101.	$\cot\left[\sum_{n=1}^{25}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right]$

$$= \cot\left[\sum_{n=1}^{25}\cot^{-1}\left(1+2\times\frac{n(n+1)}{2}\right)\right]$$

$$= \cot\left[\sum_{n=1}^{25}\cot^{-1}\left(n^2+n+1\right)\right]$$

$$= \cot\left[\sum_{n=1}^{25}\tan^{-1}\left(\frac{1}{n^2+n+1}\right)\right]$$

$$= \cot\left[\sum_{n=1}^{25}\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right)\right]$$

$$= \cot\left[\sum_{n=1}^{25}\tan^{-1}(n+1)-\tan^{-1}(n)\right]$$

$$= \cot\left[\tan^{-1}(26)-\tan^{-1}(1)\right]$$

$$= \cot\left[\tan^{-1}\frac{25}{27}\right]$$

$$= \cot\left[\cot^{-1}\frac{27}{25}\right] = \frac{27}{25}$$

$$= \frac{27}{25}$$

S.No.	Expected Answers
102.	$y = x + \dfrac{1}{x}$ $y = \dfrac{x^2+1}{x}$ $xy = x^2 + 1$ $x^2 - xy + 1 = o$ $x = \dfrac{y \pm \sqrt{y^2-4}}{2}$ $f^{-1}(y) = \dfrac{y \pm \sqrt{y^2-4}}{2}$ $f^{-1}(x) = \dfrac{x \pm \sqrt{x^2-4}}{2}$ Since range of inverse function is $[1, \infty)$, then $f^{-1}(x) = \dfrac{x+\sqrt{x^2-4}}{2}$ $f^{-1}(x) = \dfrac{x-\sqrt{x^2-4}}{2}$ then $f^{-1}(x) > 1$ is possible when, $\dfrac{x-\sqrt{x^2-4}}{2} > 1$ $x - \sqrt{x^2-4} > 2$ $(x-2)^2 > x^2 - 4$ $x^4 + 4 - 4x > x^2 - 4$ $8 > 4x$ $x < 2$ which is not possible since $x > 2$ (given). $\therefore f^{-1}(x) = \dfrac{x+\sqrt{x^2-4}}{2}$ $\dfrac{d}{dx} f^{-1}(x)$ $= \dfrac{1}{2}\left[1 + \dfrac{2x}{x\sqrt{x^2-4}}\right]$ $= \dfrac{1}{2}\dfrac{(x+\sqrt{x^2-4})}{\sqrt{x^2-4}}$

S.No.	Expected Answers
103.	$\|x^2 - 1\| = \|x^2 - 3\|$ $(x^2 - 1)^2 = (x^2 - 3)^2$ $2x\,(2x^2 - 4) = 0$ $2x^2 = 4,$ $x \neq 0 \ as \ x > 0$ $x = \pm\sqrt{2}$ But, $x = \sqrt{2} \ as \ x > 0$ We have point of intersection as $x = \sqrt{2}$ $y = \|x^2 - 1\| = x^2 - 1$ in the neighborhood of $x = \sqrt{2}$ and $y = -(x^2 - 3)$ in the neighbourhood of $x = \sqrt{2}$ Now at $x = \sqrt{2},$ $\frac{dy}{dx}$ for first curve $\frac{dy}{dx} = 2x = 2\sqrt{2}$ and $\frac{dy}{dx}$ for second curve $\frac{dy}{dx} = -2\sqrt{2}$ $\therefore \tan\theta = \left\| \frac{2\sqrt{2} - (-2\sqrt{2})}{1 + (2\sqrt{2})(-2\sqrt{2})} \right\|$ $\tan\theta = \left\| \frac{4\sqrt{2}}{-7} \right\| = \frac{4\sqrt{2}}{7}$ $\theta = \tan^{-1}\left(\frac{4\sqrt{2}}{7}\right)$

S.No.	Expected Answers
104.	(a) $y = \ln\left(\frac{x+1}{x}\right)$ $\frac{dy}{dx} = \frac{x}{x+1} \times \frac{x \times 1 - (x+1)}{x^2}$ $= \frac{-1}{x(x+1)}$ slope of tangent *at P* $m_1 = \frac{-1}{2}$ slope of normal at P $m_2 = 2$ (b) Equation of tangent at P $y - ln2 = \frac{-1}{2}(x-1)$ $2y - 2ln2 = -x + 1$ $x + 2y = 2ln2 + 1$ (c) Equation of Normal at P $y - ln2 = 2(x-4)$ $y - ln2 = 2x - 8$ $2x - y = 8 - ln2$ (d) Since the tangent at P meets x axis at A $(x_1, 0)$ Equation of tangent $x + 2y = 2ln2 + 1$ $x_1 = 2ln2 + 1$ $\therefore$ A $(2ln2 + 1, 0)$ Since, the normal meets x axis at C $(x_2, 0)$. Equation of normal $2x - y = 8 - ln2$ $2x_2 - 0 = 8 - ln2$ $x_2 = \frac{8-ln2}{2}$ $\therefore$ C $\left(\frac{8-ln2}{2}, 0\right)$

S.No.	Expected Answers
105.	The given D. E is $\frac{dy}{dx} + y\sec^2 x = \tan x \sec^2 x$ $\therefore$ I.F $= e^{\int \sec^2 x\,dx}$ $\qquad = e^{\tan x}$ So, the solution of the D. E is given by $y \cdot e^{\tan x} = \int \tan x \cdot \sec^2 x \cdot e^{\tan x}\,dx + c$ Let $\tan x = t$, so that $\sec^2 x\,dx = dt$ $y \cdot e^{\tan x} = \int t \cdot e^t\,dt + c$ $y \cdot e^{\tan x} = te^t - e^t + c$ $ye^{\tan x} = \tan x \cdot e^{\tan x} - e^{\tan x} + c$ Given $x = 0$ then $y = 1$ $\qquad c = 2$ $\therefore$ Required Solution: $y \cdot e^{\tan x} = \tan x\, e^{\tan x} - e^{\tan x} + 2$ $y = \tan x - 1 + 2e^{-\tan x}$ $\therefore y\left(\frac{\pi}{4}\right) = \frac{2}{e}$
106.	$\displaystyle\int \frac{dx}{\sqrt[4]{(x-1)^3 \cdot (x+2)^5}}$ $= \displaystyle\int \frac{dx}{\sqrt[4]{\left(\frac{x-1}{x+2}\right)^3 \cdot (x+2)^8}}$ $= \displaystyle\int \left(\frac{x-1}{x+2}\right)^{-3/4} \times \frac{1}{(x+2)^2}\,dx$ let $\frac{x-1}{x+2} = t$ $\frac{3}{(x+2)^2}\,dx = dt$ $\therefore I = \frac{1}{3}\displaystyle\int t^{-3/4}\,dt$ $= \frac{4}{3} t^{\frac{1}{4}}$ $= \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + c$

S.No.	Expected Answers
107.	We have $x^2 + 2x - 3 = (x + 3)(x - 1)$. The sign of $x^2 + 2x - 3$ for different values of x are shown. $$\|x^2 + 2x - 3\| = \begin{cases} -(x^2 + 2x - 3); if\, 0 < x < 1 \\ (x^2 + 2x - 3)\ ; if\, 1 \le x \le 2 \end{cases}$$ $\therefore I$ $$= \int_0^1 \|x^2 + 2x - 3\|\, dx + \int_1^2 \|x^2 + 2x - 3\|\, dx$$ $$= \int_0^1 -(x^2 + 2x - 3)\, dx + \int_1^2 (x^2 + 2x - 3)\, dx$$ $$= -\left[\frac{x^3}{3} + \frac{2x^2}{2} - 3x\right]_0^1 + \left[\frac{x^3}{3} + \frac{2x^2}{2} - 3x\right]_1^2$$ $$= -\left[\left(\frac{1}{3} - 1 - 3\right) - 0\right] + \left[\left(\frac{2^3}{3} + 2^2 - 3 \times 2\right) - \left(\frac{1}{3} + 1 - 3\right)\right]$$ $$= 4.$$
108.	Let the following events: G_1 - the 1st ball drawn is green. R_1 - the 1st ball drawn is red. R - the second ball drawn is red. $$P(G_1) = \frac{2}{7}, \qquad P(R_1) = \frac{5}{7}$$ $$P\left(\frac{R}{G_1}\right) = \frac{6}{7} \qquad P\left(\frac{R}{R_1}\right) = \frac{4}{7}$$ $$P(R) = P(G_1).P\left(\frac{R}{G_1}\right) + P(R_1).P\left(\frac{R}{R_1}\right)$$ $$P(R) = \frac{2}{7}.\frac{6}{7} + \frac{5}{7}.\frac{4}{7}$$ $$P(R) = \frac{32}{49}$$

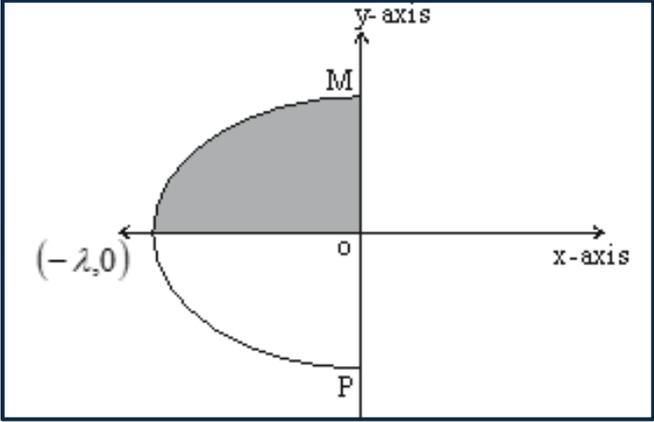| S.No.	Expected Answers
109.	(a) Let A represent obtaining sum greater than 9 and B represents black dice resulted in a 5.

$n(S) = 36$

$n(A) = \{ (4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6,6) \} = 6$

$n(B) = \{ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\} = 6$

$n(A \cap B) = \{ (5,5), (5,6) \} = 2$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{2}{36}}{\frac{6}{36}} = \frac{1}{3}.$$

(b) Let A represent obtaining sum is 4 and B represents the both the dice show different number.

$n(S) = 36,$

$n(A) = \{(1,3) (2,2), (3, 1)\} = 3$

$n(B) = 30$

$n(A \cap B) = \{(1,3), (3,1)\} = 2$

P (sum of the numbers showing different number is 4) $= P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{30}{36}} = $

$\frac{1}{15}.$

P (sum of the numbers showing different number is not 4).

$$P\left(\frac{A'}{B}\right) = 1 - P\left(\frac{A}{B}\right) = 1 - \frac{1}{15} = \frac{14}{15}$$

S.No.	Expected Answers
110.	

Let Q be the foot of the perpendicular from P to AB and let Q divide AB in the ratio k:1.

Co-ordinates of Q = $\left(\frac{3k+1}{k+1}, \frac{4k+2}{k+1}, \frac{5k+4}{k+1}\right)$

Direction Ratios of PQ = $\frac{3k+1}{k+1} - 2, \frac{4k+2}{k+1} - 1, \frac{5k+4}{k+1} - 3$

$$= \frac{k-1}{k+1}, \frac{3k+1}{k+1}, \frac{2k+1}{k+1}$$

Direction Ratios of AB = $< 3\ \text{-}1, 4 - 2, 5 - 4 >$

$$= <2, 2, 1>$$

As $PQ \perp AB$,

$$2\left(\frac{k-1}{k+1}\right) + 2\left(\frac{3k+1}{k+1}\right) + 1\left(\frac{2k+1}{k+1}\right) = 0$$

$$= -\frac{1}{10}$$

Co-ordinates of Q = $\left(\frac{7}{9}, \frac{16}{9}, \frac{35}{9}\right)$ = Location of meeting point.

S.No.	Expected Answers
111.	(a) Let $\vec{a} = -2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}, \vec{c} = 7\hat{\imath} - \hat{k}$. The vector equation of AB is given by $\vec{r} = a + \lambda(b - a)$. $\vec{r} = (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \lambda[(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})]$ $\vec{r} = (-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k}) + \lambda(3\hat{\imath} - \hat{\jmath} - 2\hat{k})$ (b) The three cities will lie on the same straight path if they are collinear. So if C lies on AB, the three points are collinear. When, $7\hat{\imath} - \hat{k} =$ $(-2\hat{\imath} + 3\hat{\jmath} + 5\hat{k})$ $+\lambda(3\hat{\imath} - \hat{\jmath} - 2\hat{k})$ we get, $7 = -2 + 3\lambda, 0$ $= 3 - \lambda, -1 = 5 - 2\lambda$ The value of $\lambda = 3$ satisfies all three equations. So, C lies on AB. Hence, we can conclude that they three cities lie on the same straight path. (c) Let P be the destination point. Co-ordinates of P is (3r – 2, -r + 3, -2r + 5). $CP = \sqrt{126} = \sqrt{(3r - 2 - 7)^2 + (3 - r)^2 + (5 - 2r + 1)^2}$ r (r – 6) = 0 r = 0 or 6 If r = 0, P is (-2, 3, 5) which is the City A. So, for r = 6, P is (16, -3, -7) Position vector of P is $16\hat{\imath} - 3\hat{\jmath} - 7\hat{k}$.

S.No.	Expected Answers
112.	(a) Requirement will be met if the lines are coplanar. To be coplanar, $$\begin{vmatrix} 2-(-1) & 4-(-3) & 6-(-5) \\ 3 & 5 & 7 \\ 1 & k & 7 \end{vmatrix} = 0$$ $$\begin{vmatrix} 3 & 7 & 11 \\ 3 & 5 & 7 \\ 1 & k & 7 \end{vmatrix} = 0$$ $3(35-7k) - 7(21-7) + 11(3k-5) = 0$ $105 - 21k - 147 + 49 + 33k - 55 = 0$ $12k = 48 \Rightarrow k = 4$ (b) The equation of the plane is: $$\begin{vmatrix} x+1 & y+3 & z+5 \\ 3 & 5 & 7 \\ 1 & 4 & 7 \end{vmatrix} = 0$$ $(x+1)(35-28) - (y+3)(21-7) + (z+5)(12-5) = 0$ $x - 2y + z = 0$.
113.	(a) $A\left(\frac{r}{p}, 0, 0\right), B(0, r, 0), C\left(0, 0, \frac{r}{p}\right)$ (b) Let the orthocentre be O (a, b, c). $OA \perp BC \Rightarrow c = bp$ $0B \perp AC \Rightarrow a = c$ $\therefore a = bp = c \Rightarrow a = \dfrac{b}{1/p} = c = k \,(say)$ Since, the orthocentre lies on the plane $px + y + pz = r$, $$pk + \frac{k}{p} + pk = r \Rightarrow k = \frac{r}{2p + \dfrac{1}{p}} = \frac{rp}{2p^2 + 1}$$ Co-ordinates of the orthocentre is $\left(\dfrac{rp}{2p^2+1}, \dfrac{r}{2p^2+1}, \dfrac{rp}{2p^2+1}\right)$.

S.No.	Expected Answers
114.	(a) Since M is the foot of the perpendicular on the ZX plane M (2, 0, 2). Since N is the foot of the perpendicular on the XY plane N (2, 1, 0). Plane through origin will be $Ax + By + Cz = 0$. Since M and N lie on the plane, we get $\frac{A}{1} = \frac{B}{-2} = \frac{C}{-1}$ The equation of plane is $x - 2y - z = 0$. (b) d.r of OP $< 2, 1, 2 >$ d.r of normal to plane OMN $< \frac{1}{2}, -1, -\frac{1}{2} >$ $$\cos(90 - \theta) = \frac{2\left(\frac{1}{2}\right) + 1(-1) + 2\left(-\frac{1}{2}\right)}{\sqrt{2^2 + 1 + 2^2}\sqrt{\left(\frac{1}{2}\right)^2 + 1 + \left(\frac{1}{2}\right)^2}} = \frac{-2}{3\sqrt{6}}$$ $\theta = \sin^{-1}\left(\frac{2}{3\sqrt{6}}\right)$ (as acute angle) (c) $\alpha = \sin^{-1}\left(\frac{2}{\sqrt{2^2 + 1 + 2^2}}\right) = \sin^{-1}\left(\frac{2}{3}\right)$ $\beta = \sin^{-1}\left(\frac{1}{\sqrt{2^2 + 1 + 2^2}}\right) = \sin^{-1}\left(\frac{1}{3}\right)$ $\gamma = \sin^{-1}\left(\frac{2}{\sqrt{2^2 + 1 + 2^2}}\right) = \sin^{-1}\left(\frac{2}{3}\right)$ R.H.S $= cosec^2\,\alpha + cosec^2\,\beta + cosec^2\,\gamma$ $= \frac{9}{4} + 9 + \frac{9}{4} = \frac{27}{2} = \left(\frac{3\sqrt{6}}{2}\right)^2 = LHS.$
115.	 ∴Required Area $= \left\| \int_{-1}^{2}[-(x + 2) - (-x^2)]dx \right\|$ $= \left\| \left[-\frac{x^2}{2} - 2x + \frac{x^3}{3} \right]_{-1}^{2} \right\|$ $= \frac{9}{2}$ sq units.

S.No.	Expected Answers
116.	The sketch of the curve: $y = 1 + \frac{8}{x^2}$ is shown in the figure:
	Required area $= \int_2^4 \left(1 + \frac{8}{x^2}\right) dx = 4$ sq units
	Now; $\int_2^a \left(1 + \frac{8}{x^2}\right) dx = 2$
	$\Rightarrow a - \dfrac{8}{a} + 2 = 2$
	$a = 2\sqrt{2}.$
117.	
	Given curve: $y^2 = 4a(x + \lambda)$ is a parabola with vertex at $(-\lambda, 0)$, axis of symmetry x-axis and it opens on the right.
	Also, when x = 0
	$\therefore y = \pm 2\sqrt{a\lambda}$
	$\therefore$ The co-ordinates of M $(0, 2\sqrt{a\lambda})$ and P $(0, -2\sqrt{a\lambda})$
	$\therefore$ Required Area $= \int_{-\lambda}^0 2\sqrt{a}\sqrt{x + \lambda}$
	$= \dfrac{4}{3}\lambda\sqrt{a\lambda}$ Squ. units.
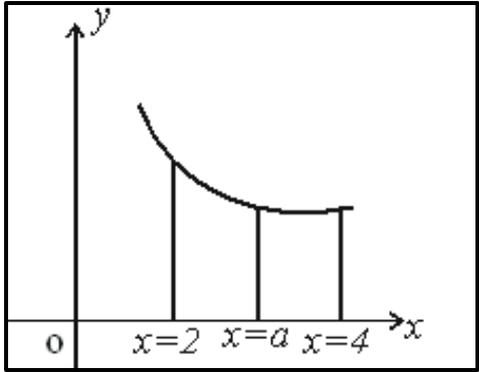
S.No.	Expected Answers
118.	$byx = r\dfrac{\sigma_y}{\sigma_x} = 0.65 \times \dfrac{10}{4}$ $\quad\quad = 1.625$ $bxy = r\dfrac{\sigma_x}{\sigma_y} = 0.65 \times \dfrac{4}{10}$ $\quad\quad = 0.26$ $y - \bar{y} = b_{yx}(x - \bar{x})$ $\Rightarrow y - 75 = 1.625(x - 15)$ $\Rightarrow y = 1.625x + 50.625$ $x - \bar{x} = b_{xy}(y - \bar{y})$ $\Rightarrow x - 15 = 0.26(y - 75)$ $\Rightarrow x = 0\cdot26x - 4.5$ (a) At x = 20, y = 83.125 (b) At y = 40, x = 5.9 hours.
119.	(a) Z is minimum at $A(0,8)$ and $Z_{min} = -32$, $\quad\quad Z$ is maximum at $E(4,0)$ and $Z_{max} = 12$. (b) At B (4, 10), Z = 4p +10 q $\quad\quad$At C (6, 8), Z = 6p + 8q $\quad\quad$As Z is maximum for both cases, 4p + 10q = 6p +8q $\quad\quad \Rightarrow$ p = q is the required condition. (c) Infinite solutions.

S.No.	Expected Answers
120.	(a)

x	y	$(x-\bar{x})^2$	$(x-\bar{x})(y-\bar{y})$
14	22	1	0
12	23	1	-1
13	17	0	0
14	24	1	2
16	18	9	-12
10	25	9	-9
13	23	0	0
12	24	1	-2

$\sum x = 104$

$\sum y = 176$

$\sum(x-\bar{x})^2 = 22$

$\sum(x-\bar{x})(y-\bar{y}) = -22$

(b)

$\bar{x} = \dfrac{104}{8} = 13,$

$\bar{y} = \dfrac{176}{8} = 22$

$\therefore b_{yx} = \dfrac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \dfrac{-22}{22} = -1$

$\therefore$ The line of regression of y on x is given by

(c) $y - 22 = -1(x - 13)$

$y = -x + 35$

Comparing $y = -x + 35$ with y = a+bx we get,

a=35 and b= -1.

S.No.	Expected Answers
121.	(a) It is clear from the graph that f is not a one-to-one function. For example, f(x)=0 corresponds to three different x values (i.e., f is many-to-one). As the graph shows, 'f' is not one-to-one. Therefore, it cannot have an inverse. (b) Use derivative to find x-coordinates of turning points: $f'(x) = 6x^2 - 6x - 12 = 0$ $\Rightarrow x^2 - x - 2 = 0$ $\Rightarrow (x + 1)(x - 2) = 0$ $\Rightarrow x = -1, 2$ $\Rightarrow a = -1 \; b = 2$ $\Rightarrow f(a) = f(-1) = 15, \; f(b) = f(2) = -12$ So, 'f' has domain $-1 \leq x \leq 2$ and range $-12 \leqslant f(x) \leqslant 15$. The domain of f $^{-1} - 12 \leq x \leq 15$ and the range is $-1 \leqslant f^{-1}(x) \leqslant 2$.
122.	P (Winning of Delhi) = P (Losing of Jaipur) = $\dfrac{1}{5}$ P (Losing of Delhi) = P (Winning of Jaipur) = $\dfrac{1}{2}$ P (Drawing of Delhi) = P (drawing of Jaipur) = $\dfrac{3}{10}$ Let X be the point of Jaipur after two games and Y be the points of Delhi after two games. According to the given information, <table><tr><td>X</td><td>10</td><td>8</td><td>6</td><td>3</td><td>0</td></tr><tr><td>Y</td><td>0</td><td>3</td><td>6</td><td>8</td><td>10</td></tr></table> (a) Now, P (X > Y) P (1ˢᵗ match win by Jaipur) P (2ⁿᵈ match win by Jaipur) + P (1ˢᵗ match win by Jaipur) P (2ⁿᵈ match draw by Jaipur) + P (1ˢᵗ match draw by Jaipur) P (2ⁿᵈ match win by Jaipur) $= \dfrac{1}{2} \cdot \dfrac{1}{2} + \dfrac{1}{2} \cdot \dfrac{3}{10} + \dfrac{3}{10} \cdot \dfrac{1}{2} = \dfrac{11}{20}$ (b) Now, P (X = Y) P (1ˢᵗ match win by Jaipur) P (2ⁿᵈ match win by Delhi) + P (1ˢᵗ match win by Delhi) P (2ⁿᵈ match win by Jaipur) + P (1ˢᵗ match draw by Jaipur) P (2ⁿᵈ match draw by Delhi) $= \dfrac{1}{2} \cdot \dfrac{1}{5} + \dfrac{1}{5} \cdot \dfrac{1}{2} + \dfrac{3}{10} \cdot \dfrac{3}{10} = \dfrac{29}{100}$

S.No.	Expected Answers
123.	 Let P, Q be the two points on their respective paths when they are closest. $\therefore$ PQ is $\perp$ to both line I and line II. General point online -I P $(\lambda+6, -2\lambda+2, 2\lambda+2)$ General point on line –II Q $(3\mu-4, -2\mu, -2\mu-1)$ d.r of PQ $< 3\mu-\lambda-10, -2\mu+2\lambda-2,$ $-2\mu-2\lambda-3>$ Line I $\perp$ PQ $\Rightarrow -3\lambda+\mu=4$ Line II $\perp$ PQ $\Rightarrow -3\lambda+17\mu=20$ Solving, $\lambda=-1, \mu=1.$ P $(5,4,0)$ Q $(-1, -2, -3)$

S.No.	Expected Answers
124.	(a) Let x and y denote, respectively, the number of Android and iOS cell phones. Thus, $x \geq 0$, $y \geq 0$. Since, the company can make at most 300 sets a week, therefore, $x+y \leq 300$.

Weekly cost (in ₹) of manufacturing the set is $1800x + 2700y$ and the company can spend up to ₹ 648000. Therefore, $1800x + 2700y \leq 648000$, i.e., or $2x + 3y \leq 720$ The total profit on x black and white sets and y colour sets is ₹ $(500x + 600y)$.

Objective function is Maximize $Z = 510x + 675y$.

(b) Values of Objective function $Z = 510x+675y$ at corner points.

C (0, 240)	162000
B (180, 120)	172800
A (300, 0)	153000
O (0,0)	0

(c) Maximum Z is 172800 at the point B (180, 120).

(d) Weekly cost = 1800x 180 + 2700 x 120 = 648000.

S.No.	Expected Answers
125.	(a) Let cost per kg of apple, grapes and orange be ₹x , ₹ y and ₹ z respectively. Then according to the given condition, we have $$4x + 3y + 2z = 600,$$ $$2x + 4y + 6z = 900,$$ $$6x + 2y + 3z = 700.$$ Which can be written in matrix form as $\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 600 \\ 900 \\ 700 \end{bmatrix}$ AX = B Where, A $= \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}$, X $= \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B $= \begin{bmatrix} 600 \\ 900 \\ 700 \end{bmatrix}$ (b) Now, $\lvert A \rvert = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 50 \neq 0$ ∴ Above system of equation is consistent and has unique solution given by X $= A^{-1}B$ Co-factor of A $= \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix} \Rightarrow adj\ (A) = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$ Now $A^{-1} = \frac{1}{\lvert A \rvert}(adjA) = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$ (c) X$= \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 600 \\ 900 \\ 700 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 2500 \\ 4000 \\ 4000 \end{bmatrix} = \begin{bmatrix} 50 \\ 80 \\ 80 \end{bmatrix}$ $x = 50, y = 80$ and $z = 80$ Hence, cost of 1 kg apple = ₹ 50, cost of 1kg grapes = ₹80 and cost of 1kg orange = ₹80.
126.	(a) <table><tr><td>**X**</td><td>299</td><td>199</td><td>99</td><td>-1</td></tr><tr><td>**P(X)**</td><td>.001</td><td>.001</td><td>.001</td><td>.997</td></tr></table> (b) Let W, denote the event of winning any money in the purchase of one ticket. Using the table: P(W) = P (299) + P (199) +P (99) =0.001 +0.001+ 0.001 = 0.003 (c) E(X) = (299)(0.001) +(199)(0.001) + (99)(0.001) +(-1)(0.997) = -0.4 Negative expectation means that a person will lose money even if multiple tickets are purchased.

S.No.	Expected Answers
127.	Consider the sample space S = {HHHH, HHHT, THHH, THHT, HHTT, TTHH…..} = $2^4 = 16$

Let X be the random variable giving Ram's gain:

According to the given information the following table summarizes the situation

Sample point	Value of x
HHHH	3
HHHT	2
THHH	2
HHTT	1
THHT	1
TTHH	1
Any other	-1

Probability distribution of variable:

X	3	2	1	−1
P(X)	$\dfrac{1}{16}$	$\dfrac{1}{8}$	$\dfrac{3}{16}$	$\dfrac{5}{8}$

Hence, $E(X) = 3.\dfrac{1}{16} + 2.\dfrac{1}{8} + 1.\dfrac{3}{16} + (-1).\dfrac{5}{8} = 0$

∴ Expectation of Ram is 0.

Since, Ram's gain and losses are Shyam's losses and gain respectively and Ram does not expect to lose or gain anything.
Shyam's expectation is also 0.

Hence, the game is fair.

S.No.	Expected Answers
128.	(a) Since the fuel cost is proportional to the square of the speed: $$C_f = kv^2$$ Given that $C_f = 75$ when v=10 $$75 = k\,(10)^2$$ **k** = 0.75. (b) Total cost = $0.75v^2 + 1000$ Cost per unit distance = $C_d = \dfrac{0.75v^2 + 1000}{v} = 0.75v + \dfrac{1000}{v}$ (c) For most economical speed cost per unit distance should be minimum. $$C_d' = 0.75 - \frac{1000}{v^2}$$ $$C_d' = 0 \Rightarrow v^2 = \frac{1000}{0.75}$$ $$v = 36 \cdot 51$$ $$C_d'' = \frac{2000}{v^3} > 0$$ v = 36.51 is point of minima. Most economical speed is 36.51 knots.
129.	$\displaystyle\int \frac{\tan\frac{x}{4}\,dx}{1 - \sin\frac{x}{4}}$ $= \displaystyle\int \frac{\sin\frac{x}{4}dx}{\cos\frac{x}{4}\left(1 - \sin\frac{x}{4}\right)}$ $= \displaystyle\int \frac{\sin\frac{x}{4}\cos\frac{x}{4}dx}{\left(1 - \sin\frac{x}{4}\right)\left(1 - \sin^2\frac{x}{4}\right)}$ Let $\sin\frac{x}{4} = t$ $\cos\frac{x}{4}\,dx = 4\,dt$ $4\displaystyle\int \frac{t\,dt}{(1+t)(1-t)^2} = -\int \frac{1}{1+t} - \int \frac{1\,dt}{1-t} + 2\int \frac{dt}{(1-t)^2}$ $= -\log(1+t) + \log(1-t) + 2\frac{1}{(1-t)} + c$ $= -\log(1 + \sin x) + \log(1 - \sin x) + 2(1 + \sin x) + c.$

S.No.	Expected Answers
130.	$\int \left(\frac{\cot x + co\, t^3\, x}{1 + \cot^3 x} \right) dx$ $= \int \frac{\cot x\, cosec^2\, x}{1 + \cot^3 x}\, dx$ let $\cot x = t$ So, $cosec^2 x\, dx = dt$ $\int \frac{t}{1 + t^3}\, dt$ $= \int \frac{t\, dt}{(1 + t)(t^2 - t + 1)}$ $= -\frac{1}{3} \int \frac{dt}{t + 1} + \frac{1}{3} \int \frac{(t + 1)}{t^2 - t + 1}\, dt$ $= -\frac{1}{3} log(t + 1) + \frac{1}{3} \int \frac{\frac{1}{2}(2t - 1) + \frac{3}{2}}{t^2 - t + 1}\, dt$ $= -\frac{1}{3} log(t + 1) + \frac{1}{6} \int \frac{(2t - 1)dt}{t^2 - t + 1} + \frac{1}{2} \int \frac{dt}{t^2 - t + 1}$ $= -\frac{1}{3} log(t + 1) + \frac{1}{6} log(t^2 - t + 1) + \frac{1}{2} \int \frac{dt}{\left(t - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$ $= -\frac{1}{3} log(t + 1) + \frac{1}{6} log(t^2 - t + 1) + \frac{1 \times 2}{2 \times \sqrt{3}} tan^{-1} \left(\frac{t - 1/2}{\sqrt{3}/2} \right)$ $= -\frac{1}{3} log(1 + \cot x) + \frac{1}{6} log(\cot^2 x - \cot x + 1) + \frac{1}{\sqrt{3}} tan^{-1} \left(\frac{2 \cot x - 1}{\sqrt{3}} \right) + c$
131.	(a) Since $f'(1)$ exists $\Rightarrow f(x)$ is differentiable at $x = 1$. $\Rightarrow f(x)$ is also continuous at $x = 1$. Hence, the statement is true. (b) $a + \sin^{-1}(1 + b) = 1$ (c) $f'(x) = \begin{cases} \frac{1}{\sqrt{1-(x+b)^2}} &, x \geq 1 \\ 1 &, x < 1 \end{cases}$ (d) $a = 1, b = -1$.

S.No.	Expected Answers
132.	(a) $2 \cos\left(x + \frac{\pi}{6}\right)$ (b) -2 (c) $\left(\frac{\pi}{3}, 2\right)$ (d) $\left(\frac{\pi}{3}, \frac{4\pi}{3}\right)$ (e) 2 (f) $\frac{1}{2}$
133.	(a) $\frac{dy}{dx} = \frac{\frac{y}{x}\left[\frac{y}{x}\tan\frac{y}{x}+1\right]}{\frac{y}{x}\tan\frac{y}{x}-1}$ (b) Appropriate Substitution: y = vx or $\frac{y}{x} = v$ (c) $\int \left(\tan v - \frac{1}{v}\right) dv = 2 \int \frac{dx}{x} + c$ (d) $\sec\left(\frac{y}{x}\right)$ =cxy or xy $\cos\frac{y}{x}$=A (e) when x=1, y=1, c= sec(1)=1.85 Solution, Sec $\left(\frac{y}{x}\right)$=1.85xy When x=1, y=1, A=cos (1)=0.54 Solution, xy cos $\left(\frac{y}{x}\right)$=0.54
134.	(a) $\tan \alpha = \frac{2}{x}$, $\tan(\alpha + \theta) = \frac{3}{x}$ (b) $\theta = \tan^{-1}\frac{3}{x} - \tan^{-1}\frac{2}{x}$ (c) $\frac{d\theta}{dx} = -\frac{(x^2-6)}{(x^2+9)(x^2+4)}$ $\frac{d\theta}{dx} = -\frac{(x-\sqrt{6})(x+\sqrt{6})}{(x^2+9)(x^2+4)}$ for slightly $< \sqrt{6}$, $\frac{d\theta}{dx} > 0$, for slightly $> \sqrt{6}$, $\frac{d\theta}{dx} < 0$ i.e. $\frac{d\theta}{dx}$ is changing its sign from +ve to - ve as x increases through $\sqrt{6}$. Hence, at $x = \sqrt{6}$ = f(x) has local maximum]. (d) $x = \pm\sqrt{6}$ (e) Sonia should stand $\sqrt{6}$ m from the wall in order to maximize her viewing angle of the painting.
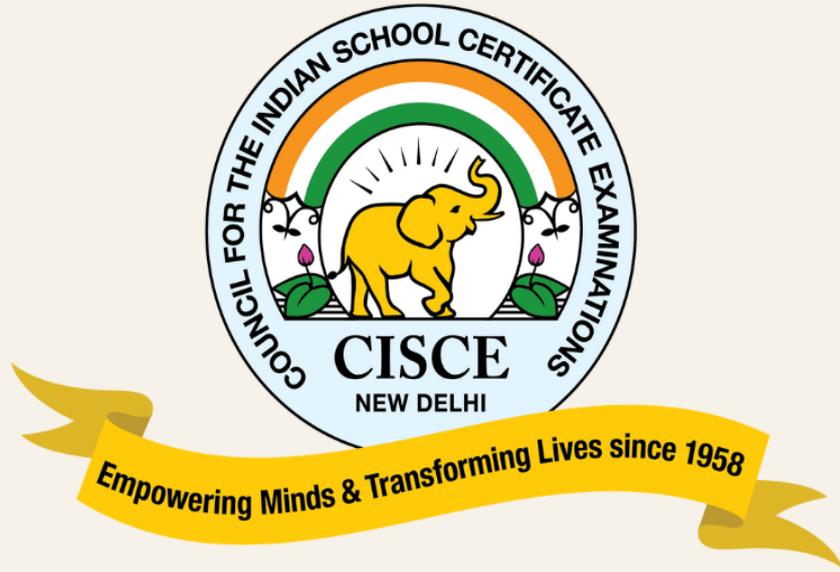
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