

MATHEMATICS

ANSWER KEY

SECTION A - 65 MARKS

Question 1

In answering Multiple Choice Questions, candidates have to write either the correct option number or the explanation against it. Please note that only ONE correct answer should be written.

- (i) (d) or a null matrix [1]
- (ii) (b) or 1 [1]
- (iii) (c) or $\frac{dy}{dx} = \frac{-y}{x}$ [1]
- (iv) (d) or Assertion is false, and Reason is true. [1]
- (v) (c) or $\frac{1}{68}$ [1]
- (vi) (a) or $\begin{pmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{pmatrix}$ [1]
- (vii) (a) or Statement 1 is true and Statement 2 is false. [1]
- (viii) (d) or 9 [1]
- (ix) (c) or Both the statements are true. [1]
- (x) (a) or Both Assertion and Reason are true and Reason is the correct explanation for Assertion. [1]
- (xi) (a) or 0.36 [1]
- (xii) $n = 3$
 $|A| = 3$.
As per question, $C = (\text{adj } A)^T$
 $\therefore |C| = |(\text{adj } A)^T| = |\text{adj } A|$
 $\therefore |C| = |A|^{n-1} = 3^{3-1} = 3^2 = 9$
 $|C^2| = |C| \cdot |C| = 9 \times 9 = 81$
Ans: 81
- (xiii) The relation is symmetric, transitive but not reflexive. [1]
- (xiv) The curve crosses x -axis at three different points. That shows, for different value of x , the value of y is zero. That is more than one domain, mapped to the same point “zero” [1]

- (xv) Two tests will be required if first machine is faulty and second is good OR both machines are faulty. [1]

$$\text{Probability that only two tests are needed} = \frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$$

$$\text{Ans: } \frac{2}{3}$$

Question 2

[2]

(i) $x = e^{\frac{x}{y}}$

$$\log x = \frac{x}{y} \log e = \frac{x}{y}$$

$$y \log x = x$$

By differentiating on both sides,

$$y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = 1$$

$$\log x \frac{dy}{dx} = 1 - \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{x - y}{x \log x}$$

- (ii) The values of 'a' for which the function $f(x) = b - ax + \sin x$ is increasing on R.

$$f'(x) = -a + \cos x \geq 0$$

It implies $a \leq \cos x$. But $-1 \leq \cos x \leq 1$

$$\text{so } a \leq -1.$$

Question 3

[2]

$$y = (x - 2)^2$$

$$\frac{dy}{dx} = 2(x - 2)$$

$$\text{Slope of the chord} = \frac{4-0}{4-2} = 2$$

Let $P(x_1, y_1)$ be the point on the curve.

Since the tangent is parallel to the chord. Therefore, their slopes are equal.

$$\therefore 2(x_1 - 2) = 2 \Rightarrow x_1 = 3 \text{ and } y_1 = 1$$

So, the point P is $(3, 1)$.

Question 4**[2]**

$\frac{dy}{dx} = y \cot 2x$, by separation of variable

$$\frac{dy}{y} = \cot 2x \, dx$$

Integrating both sides

$$\int \frac{dy}{y} = \int \cot 2x \, dx$$

$$\Rightarrow \log y = \frac{1}{2} \log |\sin 2x| + C$$

Question 5**[2]**

(i) $\int x^5 \cos(x^6) \, dx$

Let $x^6 = t$

$$6x^5 \, dx = dt$$

$$\int \cos(t) \frac{dt}{6} = \frac{1}{6} \sin t + c = \frac{1}{6} \sin(x^6) + c$$

On comparing

$$k \sin(x^6) = \frac{1}{6} \sin(x^6)$$

$$k = \frac{1}{6}$$

OR

(ii) $I = \int_0^5 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} \, dx$

Applying,

$$\int_0^a f(x) \, dx = \int_0^a f(a-x) \, dx$$

$$I = \int_0^5 \frac{\sqrt{5-x}}{\sqrt{x} + \sqrt{5-x}} \, dx$$

$$2I = \int_0^5 \frac{\sqrt{5-x} \, dx}{\sqrt{5-x} + \sqrt{x}} + \int_0^5 \frac{\sqrt{x} \, dx}{\sqrt{5-x} + \sqrt{x}}$$

$$2I = \int_0^5 1 \, dx = [x]_0^5$$

$$I = \frac{5}{2} = 2.5$$

Question 6**[2]**

$$\begin{aligned} \tan^{-1}\left(\frac{6}{10}\right) + \tan^{-1}\left(\frac{8}{10}\right) &= \tan^{-1}\frac{\frac{7}{5}}{1-\frac{12}{25}} \\ &= \tan^{-1}\left(\frac{35}{13}\right) \end{aligned}$$

Question 7**[4]**

$$\begin{aligned} \sin^{-1}(x) + \sin^{-1}(1-x) &= \cos^{-1}x \\ \Rightarrow \sin^{-1}(x) + \sin^{-1}(1-x) &= \frac{\pi}{2} - \sin^{-1}x \\ \Rightarrow \sin^{-1}(1-x) &= \frac{\pi}{2} - 2\sin^{-1}x \\ \Rightarrow (1-x) &= \sin\left(\frac{\pi}{2} - 2\sin^{-1}x\right) \\ \Rightarrow (1-x) &= \cos(2\sin^{-1}x) \\ \Rightarrow (1-x) &= \cos(\cos^{-1}(1-2x^2)) \\ \Rightarrow (1-x) &= 1-2x^2 \\ \Rightarrow 2x^2 - x &= 0 \\ \therefore x &= 0, \frac{1}{2} \end{aligned}$$

Question 8**[4]**

Given, $y = (A + Bx)e^{-2x}$

Differentiating w.r.t 'x'

$$\Rightarrow \frac{dy}{dx} = (A + Bx)(-2e^{-2x}) + e^{-2x}B$$

$$\Rightarrow \frac{dy}{dx} = (-2y) + e^{-2x}B$$

$$\Rightarrow \frac{dy}{dx} + 2y = e^{-2x}B \quad \text{(i)}$$

Differentiating w.r.t 'x'

$$\Rightarrow \frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -2e^{-2x}B \quad \text{(ii)}$$

(i) and (ii) \Rightarrow

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = -2\left(\frac{dy}{dx} + 2y\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0, \text{ Hence Proved.}$$

Question 9**[4]**

(i) Differential equation

$$x \frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$$

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\Rightarrow v + x \frac{dv}{dx} = v - \tan v$$

$$\Rightarrow \frac{dv}{\tan v} = -\frac{dx}{x}$$

Integrating both sides

$$\Rightarrow \int \cot v \, dv = -\int \frac{dx}{x}$$

$$\Rightarrow \log|\sin v| = -\log x + \log c$$

$$\Rightarrow \log\left|\sin \frac{y}{x}\right| = -\log x + \log c = \log \frac{c}{x}$$

$$\Rightarrow x \sin \frac{y}{x} = c$$

OR(ii) $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2$

$$\frac{dy}{dx} + \frac{2xy}{(1+x^2)} = \frac{4x^2}{(1+x^2)}$$

Comparing with $\frac{dy}{dx} + Py = Q$

$$P = \frac{2x}{(1+x^2)} \quad Q = \frac{4x^2}{(1+x^2)}$$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx}$$

$$\text{I.F.} = e^{\log(1+x^2)}$$

$$\text{I.F.} = (1 + x^2)$$

Thus, the solution of the differential equation is,

$$y(1 + x^2) = \int \frac{4x^2}{(1+x^2)} (1 + x^2) dx$$

$$\Rightarrow y(1 + x^2) = \frac{4x^3}{3} + c$$

Question 10**[4]**

(i) (a) $P(\text{no odd person}) = P(\text{HHH}) + P(\text{TTT}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(b) $P(\text{odd person}) = 1 - \frac{1}{4} = \frac{3}{4}$

(c) $P(\text{odd person in 4}^{\text{th}} \text{ round}) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{256}$

OR

(ii) (a) Let:

- A: student chose Mode A $\rightarrow P(A) = 0.40$

- B: student chose Mode B $\rightarrow P(B) = 0.35$

- C: student chose Mode C $\rightarrow P(C) = 0.25$

Let E: student rated the class as Excellent

Given:

- $P(E/A) = 0.20, P(E/B) = 0.30, P(E/C) = 0.50$

(b) $P(C/E)$

$$\begin{aligned}
 &= \frac{P(C) \times P(E/C)}{P(A) \times P(E/A) + P(B) \times P(E/B) + P(C) \times P(E/C)} \\
 &= \frac{0.25 \times 0.50}{(0.40 \times 0.20) + (0.35 \times 0.30) + (0.25 \times 0.50)} \\
 &= \frac{0.25 \times 0.50}{(0.08) + (0.105) + (0.125)} \\
 &= 0.403
 \end{aligned}$$

(c) There's about a 40.3% chance that a student who rated the class as "Excellent" attended Recorded lectures.

Now, numerator of, $P(A/E) = P(A) \times P(E/A) = 0.40 \times 0.20 = 0.08$
andNumerator of $P(B/E) = P(B) \times P(E/B) = 0.35 \times 0.30 = 0.105$

By checking the numerators of $P(C/E)$, $P(B/E)$ and $P(A/E)$ we observed $0.125 > 0.105 > 0.08$. Therefore, recorded lectures (Mode C) have the highest likelihood of being the chosen mode among students who gave an excellent rating.

Question 11**[6]**

- (i) Let the cost of one paper bag, one scrap book and one pastel sheet be x , y and z respectively.

$$30x + 20y + 10z = 410 \Rightarrow 3x + 2y + z = 41$$

$$20x + 10y + 20z = 290 \Rightarrow 2x + y + 2z = 29$$

$$20x + 20y + 20z = 440 \Rightarrow x + y + z = 22$$

- (ii) Given system of equations is equivalent to $AX = B$

$$\text{Where } A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix}$$

$$|A| = -2 \neq 0 \Rightarrow A^{-1} \text{ exists.}$$

$$\text{adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$$

$$\text{Thus } A^{-1} = \frac{1}{|A|} \text{adj } A = -\frac{1}{2} \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$$

$$AX = B \Rightarrow X = A^{-1}B = -\frac{1}{2} \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \\ 5 \end{bmatrix}$$

$$\therefore x = 2, y = 15, z = 5$$

- (iii) The cost of one paper bag, one scrap book and one pastel sheet be Rs 2, Rs 15 and Rs 5 respectively.

Question 12**[6]**

(i) $\int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$

$$\text{Let: } \frac{x^2 + x + 1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

Equating the coefficients of x^2 , x and constant respectively, we get

$$A + B = 1$$

$$2B + C = 1$$

$$A + 2C = 1$$

Solving for A, B, C we get,

$$C = \frac{1}{5}, B = \frac{2}{5} \text{ and } A = \frac{3}{5}$$

$$\therefore \int \frac{x^2 + x + 1}{(x+2)(x^2+1)} dx$$

$$= \int \left[\frac{3}{5(x+2)} + \frac{2x+1}{5(x^2+1)} \right] dx$$

Integrating, we get

$$= \frac{3}{5} \ln|x + 2| + \frac{1}{5} [\ln|x^2 + 1| + \tan^{-1}(x)] + C$$

OR

$$(ii) I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x}{(1 + \sin x)} dx$$

Here, $a = \frac{\pi}{4}$ and $b = \frac{3\pi}{4}$, so $a + b = \pi$.

Using, $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{(1 + \sin(\pi - x))} dx$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{\pi - x}{(1 + \sin x)} dx$$

Adding, both integrals,

$$I + I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{x + (\pi - x)}{(1 + \sin x)} dx = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{(1 + \sin x)} dx$$

$$\Rightarrow 2I = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1 - \sin x}{(1 - \sin^2 x)} dx = \pi \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\sec^2 x - \sec x \tan x) dx$$

$$\Rightarrow 2I = \pi [\tan x - \sec x]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$I = (\sqrt{2} - 1)\pi$$

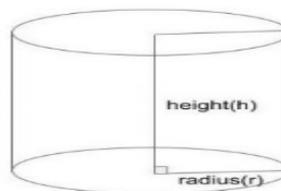
Question 13

[6]

- (i) (a) express h in terms of radius r and given volume.

$$\text{volume } V = \pi r^2 h$$

$$\frac{539}{2\pi r^2} = h$$



- (b) Let the total surface area of the closed cylinder tank be S. Expressing S in term of radius r.

$$S = 2\pi r h + 2\pi r^2$$

$$S = 2\pi r \frac{539}{2\pi r^2} + 2\pi r^2$$

$$S = 2\pi r^2 + \frac{539}{r}$$

$$(c) \frac{ds}{dr} = 4\pi r - \frac{539}{r^2}$$

Setting, $\frac{ds}{dr} = 0$ for stationary point, we get

$$539 = 4\pi r^3$$

$$r = \frac{7}{2} \text{ unit}$$

$$\text{Now, } \frac{d^2s}{dr^2} = 4\pi + \frac{539 \times 2}{r^3} > 0, \text{ when } r = \frac{7}{2} \text{ units.}$$

Therefore, the total surface area (S) of the tank is minimum when $r = \frac{7}{2}$ units.

(d) Finding the height of the tank:

$$h = \frac{539}{2\pi r^2} = \frac{539 \times 4 \times 7}{2 \times 22 \times 7 \times 7} = 7 \text{ units.}$$

OR

(ii) (a) $h(t) = \frac{1}{2}(-7t^2 + 3t + 2)$, is a polynomial function, and all polynomial functions are continuous and differentiable everywhere on R.

\therefore The function is differentiable for $t \geq 0$.

(b) Given, $h(t) = \frac{1}{2}(-7t^2 + 3t + 2)$

Differentiating w.r.t 't' we get,

$$\begin{aligned} \frac{dh}{dt} &= \frac{1}{2}(-14t + 3) \\ &= -7\left(t - \frac{3}{14}\right). \end{aligned}$$

\therefore The instantaneous rate of change of height at $t = \frac{1}{14}$

$$= \left[\frac{dh}{dt}\right]_{t=\frac{1}{14}} = \frac{3}{2} - \frac{1}{2} = 1 \text{ unit.}$$

(c) $\frac{dh}{dt} = -7\left(t - \frac{3}{14}\right)$.

Given, $t \geq 0$.

In $\left(0, \frac{3}{14}\right)$, $\frac{dh}{dt} > 0$. i.e., $h(t)$ is increasing in $\left(0, \frac{3}{14}\right)$.

$\therefore h(t)$ is increasing in $\left(-\infty, \frac{3}{14}\right)$. is false.

(d) Setting $\frac{dh}{dt} = 0$, for stationary point, we get

$$\begin{aligned} -7\left(t - \frac{3}{14}\right) &= 0 \\ \Rightarrow t &= \frac{3}{14}. \end{aligned}$$

$$\text{Now, } \frac{d^2h}{dt^2} = -7 < 0.$$

$\therefore h = f(t)$ has a local maximum at $t = \frac{3}{14}$.

\therefore Maximum height $= [h(t)]_{\frac{3}{14}} = 1.161$ units.

Question 14

[6]

(i) Let $P(E) = \frac{9}{10}, P(M) = \frac{4}{5}$ and $P(S) = \frac{7}{10}$
 $\Rightarrow P(\bar{E}) = \frac{1}{10}, P(\bar{M}) = \frac{1}{5}$ and $P(\bar{S}) = \frac{3}{10}$
 $P(X = 0) = \frac{1}{10} \times \frac{1}{5} \times \frac{3}{10} = \frac{3}{500}$
 $P(X = 1) = \left(\frac{9}{10} \times \frac{1}{5} \times \frac{3}{10}\right) + \left(\frac{1}{10} \times \frac{4}{5} \times \frac{3}{10}\right) + \left(\frac{1}{10} \times \frac{1}{5} \times \frac{7}{10}\right) = \frac{46}{500}$
 $P(X = 2) = \left(\frac{9}{10} \times \frac{4}{5} \times \frac{3}{10}\right) + \left(\frac{9}{10} \times \frac{1}{5} \times \frac{7}{10}\right) + \left(\frac{1}{10} \times \frac{4}{5} \times \frac{7}{10}\right) = \frac{199}{500}$
 $P(X = 3) = \frac{9}{10} \times \frac{4}{5} \times \frac{7}{10} = \frac{252}{500}$

(ii)

| | | | | |
|--------|-----------------|------------------|-------------------|-------------------|
| X | 0 | 1 | 2 | 3 |
| $P(X)$ | $\frac{3}{500}$ | $\frac{46}{500}$ | $\frac{199}{500}$ | $\frac{252}{500}$ |

(iii) Average number of surprise tests
 $= E(X) = 0 + \left(1 \times \frac{46}{500}\right) + \left(2 \times \frac{199}{500}\right) + \left(3 \times \frac{252}{500}\right) = \frac{1200}{500} = 2.4$

(iv) Average number of surprise tests = 2.4 > 2.3

Students are getting a good average of surprise tests. So, the current system is balanced, and the teachers do not need to change anything.

SECTION B - 15 MARKS

Question 15

[5]

(i) (a) or Statement 1 is true and Statement 2 is false.

(ii) (a) or $\vec{r} = (\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k}), \lambda \in \mathbb{R}$

$$\overrightarrow{AB} = (\hat{j} + \hat{k}) - (\hat{i} + \hat{j}) = -\hat{i} + \hat{k}$$

$$\overrightarrow{AC} = (\hat{k} + \hat{i}) - (\hat{i} + \hat{j}) = -\hat{j} + \hat{k}$$

$$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{r} = \hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} + \hat{j} + \hat{k})$$

(iii) (c) or $c = \pm\sqrt{3}$

Using, $l^2 + m^2 + n^2 = 1$

$$\Rightarrow \frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1 \Rightarrow \frac{3}{c^2} = 1$$

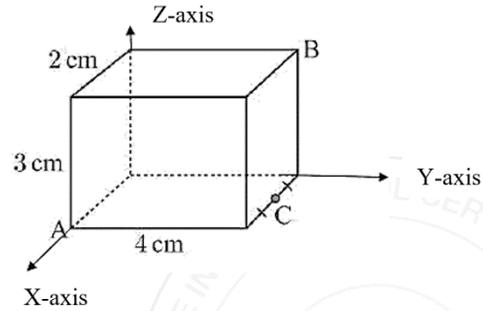
$$\Rightarrow c^2 = 3$$

$$\Rightarrow c = \pm\sqrt{3}$$

- (iv) $|\vec{a} + \vec{b}| < 1 \Rightarrow |\vec{a} + \vec{b}|^2 < 1 \Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} < 1$
 $\Rightarrow 1 + 1 + 2\vec{a} \cdot \vec{b} < 1 \Rightarrow \vec{a} \cdot \vec{b} < -\frac{1}{2} \Rightarrow |\vec{a}||\vec{b}| \cos \theta < -\frac{1}{2}$
 $\Rightarrow \cos \theta < -\frac{1}{2} \Rightarrow -1 \leq \cos \theta < -\frac{1}{2} \Rightarrow \frac{2\pi}{3} < \theta \leq \pi$ i.e., $\theta \in (\frac{2\pi}{3}, \pi]$
- (v) Placing the coordinate axes as illustrated, coordinates of A is (2,0,0), B (0,4,3) and C (1,4,0).

$$\therefore \vec{BA} = 2\hat{i} - 4\hat{j} - 3\hat{k} \text{ and } \vec{BC} = \hat{i} - 3\hat{k}$$

$$\therefore \vec{BA} \cdot \vec{BC} = 2 + 9 = 11$$



Question 16

[2]

- (i) (a) G is the centroid of ΔBCD . The coordinates are

$$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3} \right) = (3, 2, 3)$$

$$\vec{AG} = (3-0)\hat{i} + (2-1)\hat{j} + (3-2)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$$

$$|\vec{AG}| = \sqrt{3^2 + 1^2 + 1^2} = \sqrt{11} \text{ units}$$

- (b) $\vec{AB} = (3-0)\hat{i} + (0-1)\hat{j} + (1-2)\hat{k} = 3\hat{i} - \hat{j} - \hat{k}$

$$\vec{AC} = (4-0)\hat{i} + (3-1)\hat{j} + (6-2)\hat{k} = 4\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}|$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix} = -2\hat{i} - 16\hat{j} + 10\hat{k} \text{ and } |\vec{AB} \times \vec{AC}| = 6\sqrt{10}$$

$$\text{Hence area of } \Delta ABC = \frac{1}{2} |\vec{AB} \times \vec{AC}| = 3\sqrt{10} \text{ sq units}$$

OR

- (ii) Let $\vec{a} = 2x^2\hat{i} + 3x\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + x^2\hat{k}$

\therefore angle between the vectors is obtuse

$$\Rightarrow \cos \theta < 0$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} < 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} < 0$$

$$\Rightarrow 2x^2 - 6x + x^2 < 0$$

$$\Rightarrow 3x(x - 2) < 0$$

$$\Rightarrow 0 < x < 2. \text{ i.e. } x \in (0,2)$$

Question 17

[4]

- (i) Let $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$.
 $\therefore D(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$
 Dr of AD $\langle 5\lambda - 3, 2\lambda + 1, 3\lambda - 4 \rangle$ and Dr of given line L $\langle 5, 2, 3 \rangle$
 $AD \perp L$, so $5(5\lambda - 3) + 2(2\lambda + 1) + 3(3\lambda - 7) = 0 \Rightarrow \lambda = 1$
 Coordinates of D are $(2, 3, -1)$
 $AD = \sqrt{(2-0)^2 + (3-2)^2 + (-1-3)^2} = \sqrt{21}$
 Area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times 5 \times \sqrt{21} = \frac{5}{2}\sqrt{21}$ sq. units

OR

(ii) **Method-I**

Let the equation of the plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is passing through $(-1, 0, 2)$

$$\Rightarrow a(x + 1) + b(y - 0) + c(z - 2) = 0 \dots \dots \dots (1)$$

Given line $\frac{x-0}{-2} = \frac{y-1}{3} = \frac{z-1}{-1}$ passing through $(0, 1, 1)$ and having d.r. $\langle -2, 3, -1 \rangle$

Since the plane contains the line and the point
 $\Rightarrow a(1) + b(1) + c(1 - 2) = 0 \Rightarrow a + b - c = 0 \dots \dots (2)$

Also the line and normal to the plane are perpendicular
 $\Rightarrow -2a + 3b - c = 0 \dots \dots \dots (3)$

Solving (2) and (3) $\frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k$

Hence required equation of the plane is
 $\Rightarrow 2(x + 1) + 3(y - 0) + 5(z - 2) = 0$
 $\Rightarrow 2x + 3y + 5z - 8 = 0$

Method -II

Let the equation of the plane be $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$ is passing through $(-1, 0, 2)$

$$\Rightarrow a(x + 1) + b(y - 0) + c(z - 2) = 0 \dots \dots \dots (1)$$

Given line $\frac{x-0}{-2} = \frac{y-1}{3} = \frac{z-1}{-1}$ passing through $(0, 1, 1)$ and having d.r. $\langle -2, 3, -1 \rangle$

Since the plane contains the line and the point
 $\Rightarrow a(1) + b(1) + c(1 - 2) = 0 \Rightarrow a + b - c = 0 \dots \dots (2)$

Also, the line and normal to the plane are perpendicular
 $\Rightarrow -2a + 3b - c = 0 \dots \dots \dots (3)$

Hence required equation of the plane is

$$\begin{vmatrix} x + 1 & y & z - 2 \\ 1 & 1 & -1 \\ -2 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2(x + 1) + 3(y - 0) + 5(z - 2) = 0$$

$$\Rightarrow 2x + 3y + 5z - 8 = 0$$

Question 18

[4]

The required area is in two parts – one part above the x -axis and the other below the x -axis.

$$A = \int_0^4 y \, dx + \int_4^5 -y \, dx = \int_0^4 x(4 - x) \, dx + \int_4^5 x(x - 4) \, dx$$

$$= \left[2x^2 - \frac{x^3}{3} \right]_0^4 + \left[\frac{x^3}{3} - 2x^2 \right]_4^5 = \left(32 - \frac{64}{3} \right) + \left(\frac{125}{3} - 50 \right) - \left(\frac{64}{3} - 32 \right)$$

$$= \frac{32}{3} - \frac{25}{3} + \frac{32}{3} = 13 \text{ sq units}$$

SECTION C - 15 MARKS

Question 19

[5]

(i) (b) or $MC = AC$

(ii) (b) or Both statements are true but Statement 2 is not the correct explanation of Statement 1.

(iii) Given: $\bar{x} = 53$, $\bar{y} = 28$, $b_{yx} = -1.2$, $b_{xy} = -0.3$

$$\text{Using, } r^2 = b_{yx} \times b_{xy}$$

$$= -1.2 \times -0.3 = 0.36$$

$$\Rightarrow r = -0.6$$

Ans : -0.6

(iv) Given, $R(x) = 36x + 3x^2 + 5$

$$\Rightarrow MR = \frac{d}{dx} (36x + 3x^2 + 5)$$

$$= 36 + 6x$$

$$\therefore MR(x = 5) = 66$$

Ans: 66

- (v) Given $C(x) = 210x + 7000$, $R(x) = 280x$
 Minimum number must be sold daily when $R(x) = C(x)$
 $\Rightarrow 210x + 7000 = 280x$
 $\Rightarrow 70x = 7000$
 $\Rightarrow x = 100.$
 \therefore Minimum number that must be sold is 100.
 Ans: 100

Question 20

[2]

(i) $C(x) = 4000 + 14x - 0.04x^2$

$\Rightarrow C'(x) = 14 - 0.08x$

Now, $C'(x) = 0 \Rightarrow 14 = 0.08x \Rightarrow x = 175$

$C''(x) = -0.08 < 0$

$\therefore C(x)$ will be maximum at $x = 175.$

As per question, we have to minimise the maintenance cost.

Since x can be 0 to 500 apartments,

$C(0) = 4000 + 14 \times 0 - 0.04 \times (0)^2 = 4000$

$C(500) = 4000 + 14 \times 500 - 0.04 \times (500)^2 = 1000$

\therefore The complex must have 500 apartments to minimise the maintenance cost.

OR

(ii) Given, $x = 100 - 4p$

$\Rightarrow p = \frac{100 - x}{4}$

$\therefore R(x) = px = \frac{100x - x^2}{4}$

$\Rightarrow MR = \frac{d}{dx} \left(\frac{100x - x^2}{4} \right) = \frac{100 - 2x}{4}$

$\therefore MR = 0$

$\therefore x = 50$

Question 21

[4]

$\bar{x} = \frac{\sum x}{n} = 170, \quad \bar{y} = \frac{\sum y}{n} = 192,$

$b_{yx} = r \frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{20}{60} = 0.2$

Regression equation y on x is

$y - 192 = 0.2(x - 170)$

$$\Rightarrow y - 192 = 0.2x - 34$$

$$\Rightarrow y = 0.2x + 158$$

Putting, $x = 200$

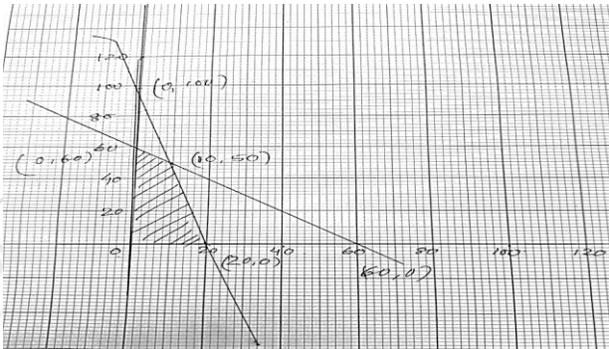
$$\Rightarrow y = 0.2(200) + 158 = 198$$

\therefore Expenditure on food and entertainment = ₹ 198.

Question 22

[4]

- (i) (a) From graph, corner points are: A (0, 60), B (10, 50), C (20, 0), D (0, 0)



- (b) $Z = px + qy$
Given, Z is maximum at (0, 60) and (10, 50)

$$\therefore 0 \cdot p + 60 \cdot q = 10 \cdot p + 50 \cdot q$$

$$\Rightarrow 10p = 10q$$

$$\Rightarrow p = q$$

So, there can be infinite number of optimal solutions.

OR

- (ii) (a) Equation of line AD:

$$\frac{x}{25} + \frac{y}{20} = 1 \Rightarrow 2x + y = 50$$

Equation of line EC:

$$\frac{x}{40} + \frac{y}{20} = 1 \Rightarrow x + 2y = 40$$

As origin lies in the region, $2x + y \leq 50$ and $x + 2y \leq 40$

Therefore, the constraints are,

$$2x + y \leq 50$$

$$x + 2y \leq 40$$

$$x \geq 0, y \geq 0$$

- (b) Solving simultaneously, the coordinate of B is (20, 10).

- (c) The corner points are $O(0,0)$, $C(0,20)$, $B(20,10)$ and $A(25,0)$.

At O, $Z = 0$

At C, $Z = 20$

At B, $Z = 30$

At A, $Z = 25$

\therefore Maximum value of Z is 30.