

# MATHEMATICS

## ANSWER KEY

### SECTION A - 65 MARKS

#### Question 1

In answering Multiple Choice Questions, candidates have to write either the correct option number or the explanation against it. Please note that only ONE correct answer should be written.

- (i) (d) a null matrix [1]
- (ii) (b)  $\frac{a^x \cdot e^x}{\log_e(ae)} + c$  [1]
- (iii) (c)  $|a| < \frac{1}{\sqrt{3}}$  [1]
- (iv) (d) Assertion is false and Reason is true. [1]
- (v) (c)  $\frac{1}{68}$  [1]
- (vi) (d)  $6k$  [1]
- (vii) (a) Statement 1 is true, and Statement 2 is false. [1]
- (viii) (b) 0 [1]
- (ix) (c) Statement 1 is true only if Statement 2 is true. [1]
- (x) (a) Both Assertion and Reason are true and Reason is the correct explanation for Assertion. [1]
- (xi) (a) 0.36 [1]
- (xii)  $n = 3$  [1]  
 $|A| = 3.$   
As per question,  $C = (\text{adj } A)^T$   
 $\therefore |C| = |(\text{adj } A)^T| = |\text{adj } A|$   
 $\therefore |C| = |A|^{n-1} = 3^{3-1} = 3^2 = 9$   
 $|C^2| = |C| \cdot |C| = 9 \times 9 = 81$
- ANS : 81**
- (xiii) The relation is symmetric, transitive but not reflexive. [1]
- (xiv) Since each line in co-domain of the function parallel to x-axis, doesn't cut the graph of function at least one point, therefore the  $f(x)$  is not an onto function. [1]

- (xv) Two tests will be required if first machine is faulty and second is good OR both machines are faulty. [1]

$$\text{Probability that only two tests are needed} = \frac{2}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} = \frac{2}{3}$$

$$\text{ANS: } \frac{2}{3}$$

### Question 2

[2]

(i)  $x^y = y^x$

$$y \log x = x \log y$$

By differentiating on both sides,

$$y \cdot \frac{1}{x} + \log x \frac{dy}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \log y$$

$$\frac{dy}{dx} \left( \log x - \frac{x}{y} \right) = \log y - \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\log y - \frac{y}{x}}{\log x - \frac{x}{y}}$$

OR

(ii)  $f(x) = x^2 e^{-x}$

$$\frac{dy}{dx} = -x^2 e^{-x} + e^{-x} \cdot 2x$$

$$= \frac{1}{e^x} (2x - x^2) = \frac{1}{e^x} (2 - x)x = -e^{-x} [x(x - 2)]$$

$\therefore e^x$  is always positive

$$\therefore \text{Setting } x(x - 2) = 0 \Rightarrow x = 0 \text{ or } x = 2$$

For strictly decreasing function  $\frac{dy}{dx} < 0 \Rightarrow x < 0 \text{ or } x > 2$   
 $\Rightarrow x \in (-\infty, 0) \cup (2, \infty)$

For strictly increasing function  $\frac{dy}{dx} > 0 \Rightarrow 0 < x < 2 \Rightarrow x \in (0, 2)$

$\therefore f(x)$  is increasing in  $(0, 2)$  and decreasing in  $(-\infty, 0) \cup (2, \infty)$

### Question 3

[2]

$$\int_0^{\sqrt{2}} [x^2] dx$$

We know greatest integer function is discontinuous when  $x$  is an integer.

$$\therefore \int_0^{\sqrt{2}} [x^2] dx = \int_0^1 0 dx + \int_1^{\sqrt{2}} 1 dx = x \Big|_1^{\sqrt{2}} = \sqrt{2} - 1.$$

**Question 4****[2]**

$$y = 4x^2 - 2x^3$$

$$\frac{dy}{dx} = 8x - 6x^2$$

Let  $P(x_1, y_1)$  be the point of contact.

$$\therefore \text{Slope of the tangents} = 8x_1 - 6x_1^2$$

$$\text{Tangent equation: } y - y_1 = (8x_1 - 6x_1^2)(x - x_1)$$

$\therefore$  the tangent passes through the origin

$$y_1 = (8x_1 - 6x_1^2)x_1$$

$$\text{Now, } y_1 = 4x_1^2 - 2x_1^3 = 8x_1^2 - 6x_1^3$$

$$\Rightarrow 4 - 2x_1 = 8 - 6x_1$$

$$\Rightarrow 4x_1 = 4$$

$$x_1 = 1 \Rightarrow y_1 = 2$$

The point on the curve is (1,2)

**Question 5****[2]**

$$(i) \int \frac{2x^3 - 1}{x^4 + x} dx$$

Dividing both nominator and denominator by ' $x$ ' we get

$$= \int \frac{2x - \frac{1}{x^2}}{x^2 + \frac{1}{x}} dx$$

$$\text{Let } x^2 + \frac{1}{x} = t \Rightarrow \left(2x - \frac{1}{x^2}\right) dx = dt$$

$$= \int \frac{dt}{t}$$

$$= \log_e |t| + c$$

$$= \log_e \left| x^2 + \frac{1}{x} \right| + c$$

**OR**

$$(ii) \int e^x \operatorname{cosec} x (1 - \cot x) dx$$

$$= \int e^x (\operatorname{cosec} x - \operatorname{cosec} x \cdot \cot x) dx$$

$$= \int e^x (\operatorname{cosec} x + (-\operatorname{cosec} x \cdot \cot x)) dx$$

$$[\because \int e^x (f(x) + f'(x)) dx = e^x f(x) + c]$$

$$= e^x \operatorname{cosec} x + c$$

**Question 6**

[2]

$$\begin{aligned} \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y-x}{y+x}\right) &= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{1-\frac{x}{y}}{1+\frac{x}{y}}\right) \\ &= \tan^{-1}\left(\frac{x}{y}\right) + \tan^{-1}(1) - \tan^{-1}\left(\frac{x}{y}\right) \\ &= \tan^{-1}(1) = \frac{\pi}{4} \end{aligned}$$

**Question 7**

[4]

$$\begin{aligned} \sin^{-1}(x) + \sin^{-1}(1-x) &= \cos^{-1} x \\ \Rightarrow \sin^{-1}(x) + \sin^{-1}(1-x) &= \frac{\pi}{2} - \sin^{-1} x \\ \Rightarrow \sin^{-1}(1-x) &= \frac{\pi}{2} - 2 \sin^{-1} x \\ \Rightarrow (1-x) &= \sin\left(\frac{\pi}{2} - 2 \sin^{-1} x\right) \\ \Rightarrow (1-x) &= \cos(2 \sin^{-1} x) \\ \Rightarrow (1-x) &= \cos(\cos^{-1}(1-2x^2)) \\ \Rightarrow (1-x) &= 1-2x^2 \\ \Rightarrow 2x^2 - x &= 0 \\ \therefore x &= 0, \frac{1}{2} \end{aligned}$$

**Question 8**

[4]

$$I = \int \sqrt{\sec \frac{x}{2} - 1} dx = \int \sqrt{\frac{1 - \cos \frac{x}{2}}{\cos \frac{x}{2}}} dx = \int \sqrt{\frac{(1 - \cos \frac{x}{2})(1 + \cos \frac{x}{2})}{\cos \frac{x}{2}(1 + \cos \frac{x}{2})}} dx = \int \frac{\sin \frac{x}{2}}{\sqrt{\cos^2 \frac{x}{2} + \cos \frac{x}{2}}} dx$$

$$\text{Let } \cos \frac{x}{2} = t \Rightarrow -\sin \frac{x}{2} \cdot \frac{1}{2} dx = dt \Rightarrow \sin \frac{x}{2} \cdot dx = -2dt$$

$$\therefore I = -2 \int \frac{dt}{\sqrt{t^2 + t}} = -2 \int \frac{dt}{\sqrt{\left(t + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2}} = -2 \log_e \left| \left(t + \frac{1}{2}\right) + \sqrt{t^2 + t} \right| + c$$

$$= -2 \log_e \left| \left(\cos \frac{x}{2} + \frac{1}{2}\right) + \sqrt{\cos^2 \frac{x}{2} + \cos \frac{x}{2}} \right| + c$$

**Question 9**

[4]

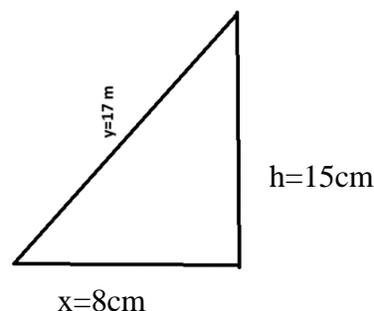
(i)  $x^2 + h^2 = y^2 \Rightarrow x^2 + 225 = y^2$

Given,  $\frac{dy}{dt} = 1 \text{ m/sec}$

Differentiating w.r.t 't'

$$2x \frac{dx}{dt} = 2y \frac{dy}{dt}$$

$$8 \times \frac{dx}{dt} = 17 \times 1,$$



**OR**

(ii) **Method I**  $y = [x + \sqrt{a^2 + x^2}]^m$

$$\frac{dy}{dx} = \frac{m [x + \sqrt{a^2 + x^2}]^{m-1}}{x + \sqrt{a^2 + x^2}} \cdot \left[ 1 + \frac{1 \cdot 2x}{2\sqrt{a^2 + x^2}} \right] = my \frac{1}{\sqrt{a^2 + x^2}}$$

$$\sqrt{a^2 + x^2} \frac{dy}{dx} = my$$

$$\sqrt{a^2 + x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{2} \cdot \frac{2x}{\sqrt{a^2 + x^2}} = m \frac{dy}{dx}$$

$$(a^2 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - m\sqrt{a^2 + x^2} \frac{dy}{dx} = 0$$

$$(a^2 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - m^2y = 0$$

**Method II**  $y = [x + \sqrt{a^2 + x^2}]^m$

$$\frac{dy}{dx} = \frac{m [x + \sqrt{a^2 + x^2}]^{m-1}}{x + \sqrt{a^2 + x^2}} \cdot \left[ 1 + \frac{1 \cdot 2x}{2\sqrt{a^2 + x^2}} \right] = my \frac{1}{\sqrt{a^2 + x^2}}$$

$$\sqrt{a^2 + x^2} \frac{dy}{dx} = my$$

Squaring both sides we get,

$$(a^2 + x^2) \left( \frac{dy}{dx} \right)^2 = m^2y^2$$

Differentiating w.r.t 'x' ,

$$2x \left( \frac{dy}{dx} \right)^2 + (a^2 + x^2) 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = 2m^2y \frac{dy}{dx}$$

$$\Rightarrow (a^2 + x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - m^2y = 0$$

**Question 10****[4]**

(i)

$$(a) \quad P(\text{no odd person}) = P(\text{HHH}) + P(\text{TTT}) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$(b) \quad P(\text{odd person}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$(c) \quad P(\text{odd person in 4}^{\text{th}} \text{ round}) = \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{3}{256}$$

**OR**

(ii)

(a) Since events A, B, C, D are equally likely

$$\Rightarrow P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$$

$$\text{As per question, } P(L/A) = \frac{24}{100}, P(L/B) = \frac{22}{100}, P(L/C) = \frac{17}{100}, P(L/D) = \frac{9}{100}$$

The probability that a randomly selected child is left-handed=

$$P(A) \times P(L/A) + P(B) \times P(L/B) + P(C) \times P(L/C) + P(D) \times P(L/D)$$

$$\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}$$

(b) The probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed

$$= P(L/B) + P(L/C)$$

$$= \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$$

(c)  $P(A/L)$ 

$$= \frac{P(A) \times P(L/A)}{P(A) \times P(L/A) + P(B) \times P(L/B) + P(C) \times P(L/C) + P(D) \times P(L/D)}$$

$$= \frac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}} = \frac{1}{3}$$

**Question 11****[6]**

(i) Let the cost of one paper bag, one scrap book and one pastel sheet be x, y and z respectively.

$$30x + 20y + 10z = 410 \Rightarrow 3x + 2y + z = 41$$

$$20x + 10y + 20z = 290 \Rightarrow 2x + y + 2z = 29$$

$$20x + 20y + 20z = 440 \Rightarrow x + y + z = 22$$

(ii) Given system of equations is equivalent to  $AX = B$

Where  $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ ,  $B = \begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix}$

$|A| = -2 \neq 0 \Rightarrow A^{-1}$  exists.

$\text{adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$

Thus  $A^{-1} = \frac{1}{|A|} \text{adj } A = -\frac{1}{2} \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B = -\frac{1}{2} \begin{bmatrix} -1 & -1 & 3 \\ 0 & 2 & -4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 41 \\ 29 \\ 22 \end{bmatrix} = \begin{bmatrix} 2 \\ 15 \\ 5 \end{bmatrix}$

$\therefore x = 2, y = 15, z = 5$

- (iii) The cost of one paper bag, one scrap book and one pastel sheet be Rs 2, Rs 15 and Rs 5 respectively.

**Question 12**

[6]

(i) **Method-I**

$(xdy - ydx) y \sin\left(\frac{y}{x}\right) = (ydx + xdy) x \cos\left(\frac{y}{x}\right)$ .

$\Rightarrow (xdy - ydx) \frac{\sin\left(\frac{y}{x}\right)}{\cos\left(\frac{y}{x}\right)} = \frac{(ydx + xdy) x}{y}$

$\Rightarrow \frac{(xdy - ydx)}{x^2} \tan\left(\frac{y}{x}\right) = \frac{(ydx + xdy) x}{x^2 y}$

$\Rightarrow \frac{(xdy - ydx)}{x^2} \tan\left(\frac{y}{x}\right) = \frac{(ydx + xdy)}{xy}$

$\Rightarrow \tan\left(\frac{y}{x}\right) \cdot d\left(\frac{y}{x}\right) = \frac{d(xy)}{xy}$ ,

which is variable separable form where variables are  $\frac{y}{x}$  and  $xy$

Integrating both sides,

$\Rightarrow \int \tan\frac{y}{x} \cdot d\left(\frac{y}{x}\right) = \int \frac{d(xy)}{xy}$

$\Rightarrow \log \left| \sec\left(\frac{y}{x}\right) \right| = \log(xy) + \log c$

$\Rightarrow \sec\left(\frac{y}{x}\right) = c(xy)$ .

Given, when  $x = 1, y = \pi \Rightarrow c = -\frac{1}{\pi}$ .

Hence required particular solution is:  $\sec\left(\frac{y}{x}\right) = -\frac{1}{\pi}(xy)$ .

**Method II**

$(xdy - ydx) y \sin\left(\frac{y}{x}\right) = (ydx + xdy) x \cos\left(\frac{y}{x}\right)$ .

$\Rightarrow \frac{dy}{dx} = \frac{(xy) \cos\left(\frac{y}{x}\right) + y^2 \sin\left(\frac{y}{x}\right)}{(xy) \sin\left(\frac{y}{x}\right) - x^2 \cos\left(\frac{y}{x}\right)}$

$$\Rightarrow \frac{dy}{dx} = \frac{1 + \left(\frac{y}{x}\right) \tan\left(\frac{y}{x}\right)}{\tan\left(\frac{y}{x}\right) - \frac{1}{\left(\frac{y}{x}\right)}}$$

which is homogenous differential equation of type  $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ .

$$\text{Let } y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$\Rightarrow v + x \frac{dv}{dx} = \frac{1 + v \tan v}{\tan v - \frac{1}{v}}$$

$$\Rightarrow \left(\tan v - \frac{1}{v}\right) dv = 2 \frac{dx}{x},$$

which is variable separable form where variables are  $v$  and  $x$

Integrating both sides,

$$\Rightarrow \int \left(\tan v - \frac{1}{v}\right) dv = 2 \int \frac{dx}{x}$$

$$\Rightarrow \log|\sec v| - \log|v| = 2 \log|x| + \log c$$

$$\Rightarrow \log|\sec v| = \log|v \cdot x^2 \cdot c|$$

$$\Rightarrow \sec v = c(xy)$$

$$\Rightarrow \sec\left(\frac{y}{x}\right) = c(xy).$$

$$\text{Given, when } x = 1, y = \pi \Rightarrow c = -\frac{1}{\pi}.$$

$$\text{Hence required particular solution is: } \sec\left(\frac{y}{x}\right) = -\frac{1}{\pi}(xy).$$

**OR**

$$(ii) \int_0^\pi (\sin^4 x + \cos^4 x) dx \quad [ \because \sin^4 x + \cos^4 x = 1 - \frac{1}{2} \sin^2 2x = \frac{1}{4} (3 + \cos 4x) ]$$

$$= \frac{1}{4} \int_0^\pi (3 + \cos 4x) dx$$

$$= \frac{1}{4} \left[ 3x + \frac{\sin 4x}{4} \right]_0^\pi = \frac{3\pi}{4}$$

$$I = \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1 + e^x} dx$$

$$= \int_{-2\pi}^{2\pi} \frac{\sin^4(-x) + \cos^4(-x)}{1 + e^{-x}} dx \quad [ \because \int_a^b f(x) dx = \int_a^b f(a+b-x) dx ]$$

$$= \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1 + e^{-x}} dx$$

$$= \int_{-2\pi}^{2\pi} \frac{e^x(\sin^4 x + \cos^4 x)}{1 + e^x} dx$$

$$I + I = \int_{-2\pi}^{2\pi} \frac{\sin^4 x + \cos^4 x}{1 + e^x} dx + \int_{-2\pi}^{2\pi} \frac{e^x(\sin^4 x + \cos^4 x)}{1 + e^x} dx$$

$$\therefore 2I = \int_{-2\pi}^{2\pi} (\sin^4 x + \cos^4 x) dx$$

$$= 2 \int_0^{2\pi} (\sin^4 x + \cos^4 x) dx$$

$$[ \because \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx \text{ if } f(-x) = f(x) ]$$

$$\therefore I = \int_0^{2\pi} (\sin^4 x + \cos^4 x) dx$$

$$= 2 \int_0^\pi (\sin^4 x + \cos^4 x) dx$$

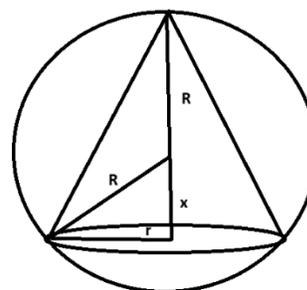
$$[ \because \int_0^{2a} f(x) dx = 2 \int_0^a f(x) dx \text{ if } f(2a-x) = f(x) ]$$

$$= 2 \cdot \frac{3\pi}{4} = \frac{3\pi}{2}$$

**Question 13**

[6]

- (i) Let the given radius of sphere be  $R$ .  
Consider radius of cone be  $r$  and height be  $h = R + x$



**Method -I**

From the diagram  $r^2 + x^2 = R^2$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 (R + x) = \frac{1}{3}\pi (R + x)(R^2 - x^2)$$

$$\frac{dV}{dx} = \frac{1}{3}\pi [(R^2 - x^2)(1) + (R + x)(-2x)]$$

Setting,  $\frac{dV}{dx} = 0$

$$\Rightarrow \frac{1}{3}\pi [(R^2 - x^2)(1) + (R + x)(-2x)] = 0$$

$$\Rightarrow (R + x)(R - 3x) = 0$$

$$\Rightarrow R = 3x \text{ or } x = \frac{R}{3}$$

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi [-2R - 6x] < 0 \text{ at } x = \frac{R}{3}$$

Volume is maximum at  $x = \frac{R}{3}$

$$\text{Height of cone} = R + x = R + \frac{R}{3} = \frac{4R}{3}$$

$$\text{Ratio of height of cone to diameter of sphere} = \frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$$

**Method -II**

From the diagram  $R^2 = r^2 + x^2 = r^2 + (h - R)^2$

$$\therefore R^2 = r^2 + h^2 + R^2 - 2Rh$$

$$\therefore r^2 = 2Rh - h^2$$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi (2Rh - h^2)h = \frac{1}{3}\pi (2Rh^2 - h^3)$$

$$\therefore \frac{dV}{dh} = \frac{1}{3}\pi (4Rh - 3h^2)$$

Setting  $\frac{dV}{dh} = 0$ , we get  $h = \frac{4R}{3}$

$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi (4R - 6h) = \frac{1}{3}\pi (4R - 8R) < 0$$

$\therefore$  Volume is maximum at  $h = \frac{4R}{3}$

$$\text{Ratio of height of cone to diameter of sphere} = \frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$$

$$\text{Setting } \frac{dV}{dh} = 0, \text{ we get } h = \frac{4R}{3}$$

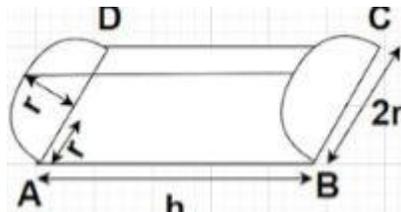
$$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R - 6h) = \frac{1}{3}\pi(4R - 8R) < 0$$

$$\therefore \text{Volume is maximum at } h = \frac{4R}{3}$$

$$\text{Ratio of height of cone to diameter of sphere} = \frac{h}{2R} = \frac{\frac{4R}{3}}{2R} = \frac{2}{3}$$

OR

(ii)



$$\text{Volume of half cylinder} = \frac{1}{2}\pi r^2 h = V \Rightarrow h = \frac{2V}{\pi r^2}$$

$$S = \pi r^2 + \pi r h + 2r h = \pi r^2 + \pi r \cdot \frac{2V}{\pi r^2} + 2r \cdot \frac{2V}{\pi r^2} = \pi r^2 + \frac{1}{r} \left( 2V + \frac{4V}{\pi} \right)$$

$$\frac{dS}{dr} = 2\pi r - \frac{1}{r^2} \left( 2V + \frac{4V}{\pi} \right)$$

$$\text{Setting } \frac{dS}{dr} = 0 \Rightarrow 2\pi r - \frac{1}{r^2} \left( 2V + \frac{4V}{\pi} \right) = 0$$

$$\Rightarrow 2\pi r = \frac{1}{r^2} \left( 2V + \frac{4V}{\pi} \right) \Rightarrow \pi r^3 = V + \frac{2V}{\pi} \Rightarrow \pi r^3 = V \left( \frac{\pi + 2}{\pi} \right)$$

$$\pi r^3 = \frac{1}{2}\pi r^2 h \cdot \frac{\pi + 2}{\pi} \Rightarrow \frac{2r}{h} = \frac{\pi + 2}{\pi}$$

$$\therefore \text{Ratio of the length of cylinder to the diameter of semi-circular ends} = \frac{\pi}{\pi + 2}$$

$$\frac{d^2S}{dr^2} = 2\pi + \frac{2}{r^3} \left( 2V + \frac{4V}{\pi} \right) > 0$$

$\therefore$  Surface area is minimum.

**Question 14**

[6]

(i)  $X = -3, -1, 0, 1$

$$(ii) \quad P(X = -3) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} = \frac{125}{216}$$

$$P(X = -1) = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{25}{216}$$

$$P(X = 0) = \frac{5}{6} \times \frac{1}{6} = \frac{5}{36}$$

$$P(X = 1) = \frac{1}{6}$$

Required probability distribution :

<b>X</b>	-3	-1	0	1
<b>P(X)</b>	$\frac{125}{216}$	$\frac{25}{216}$	$\frac{5}{36}$	$\frac{1}{6}$

$$(iii) \quad \text{Expected value} = \sum px = (-3) \left( \frac{125}{216} \right) + (-1) \left( \frac{25}{216} \right) + 0 + \left( \frac{1}{6} \right)$$

$$= -\frac{91}{54} = -1 \frac{37}{54} \approx -1.69$$

### SECTION B - 15 MARKS

#### Question 15

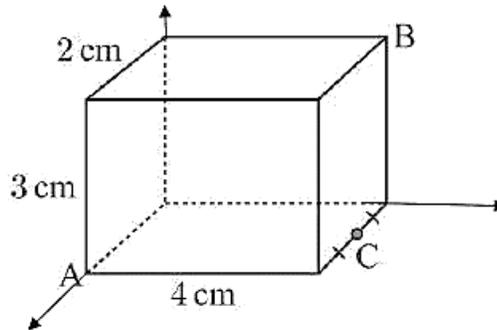
[5]

(i) (a) 1 only

(ii) (c)  $\frac{7}{5}$

(iii) As per question  $|\vec{a}| = 1$ ,  $\vec{a} \cdot \vec{b} = 0$   
 $\therefore (\vec{a} + 2\vec{b}) \cdot (3\vec{a} - \vec{b}) = -5$   
 $\Rightarrow 3|\vec{a}|^2 + 5(\vec{a} \cdot \vec{b}) - 2|\vec{b}|^2 = -5$   
 $\Rightarrow 2|\vec{b}|^2 = 8 \Rightarrow |\vec{b}| = 2$

(iv) Placing the coordinate axes as illustrated, coordinates of A is (2,0,0), B (0,4,3) and C (1,4,0).



$$\therefore \vec{BA} = 2\hat{i} - 4\hat{j} - 3\hat{k} \text{ and } \vec{BC} = \hat{i} - 3\hat{k}$$

$$\therefore \vec{BA} \cdot \vec{BC} = 2 + 9 = 11$$

(v) Using,  $l^2 + m^2 + n^2 = 1$   
 $\Rightarrow \frac{1}{c^2} + \frac{1}{c^2} + \frac{1}{c^2} = 1 \Rightarrow \frac{3}{c^2} = 1$   
 $\Rightarrow c^2 = 3$   
 $\Rightarrow c = \pm\sqrt{3}$

**Question 16**

[2]

- (i) A vector of magnitude 9 units and perpendicular to the vectors  $\vec{a}$  and  $\vec{b}$  is  $9\left(\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}\right)$

Given,  $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = -2\hat{i} + \hat{j} - 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 1 \\ -2 & 1 & -2 \end{vmatrix} = \hat{i} + 6\hat{j} + 2\hat{k} \text{ and } |\vec{a} \times \vec{b}| = \sqrt{41}$$

Hence required vector is  $\frac{9}{\sqrt{41}}(\hat{i} + 6\hat{j} + 2\hat{k})$

**OR**

- (ii) Let  $\vec{a} = 2x^2\hat{i} + 3x\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + x^2\hat{k}$

∴ angle between the vectors is obtuse

$$\Rightarrow \cos \theta < 0$$

$$\Rightarrow \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} < 0$$

$$\Rightarrow \vec{a} \cdot \vec{b} < 0$$

$$\Rightarrow 2x^2 - 6x + x^2 < 0$$

$$\Rightarrow 3x(x - 2) < 0$$

$$\Rightarrow 0 < x < 2. \text{ i.e. } x \in (0, 2)$$

**Question 17**

[4]

- (i) P.V. of the point on the line  $2\hat{i} - 2\hat{j} + 3\hat{k}$

Direction vector  $\hat{i} - \hat{j} + 4\hat{k}$

Given plane  $\vec{r} \cdot (\hat{i} + 5\hat{j} + \hat{k}) - 5 = 0$

∴ normal vector of the plane  $\hat{i} + 5\hat{j} + \hat{k}$

If straight line and plane are parallel to each other

⇒ Direction vector of line and normal vector of the plane are perpendicular to each other

Now we get  $(\hat{i} - \hat{j} + 4\hat{k})(\hat{i} + 5\hat{j} + \hat{k}) = 0$

∴ Since the dot product is zero, they are perpendicular to each other.

⇒ straight line and plane are parallel to each other.

$$\text{Required distance} = \frac{(2\hat{i}-2\hat{j}+3\hat{k}) \cdot (i+5j+k) - 5}{\sqrt{1+25+1}} = \frac{10}{\sqrt{27}} = \frac{10}{3\sqrt{3}} \text{ units.}$$

**OR**

(ii) **Method-I**

Let the equation of the plane be  $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$  is passing through  $(-1, 0, 2)$

$$\Rightarrow a(x + 1) + b(y - 0) + c(z - 2) = 0 \dots\dots\dots(1)$$

Given line  $\frac{x-0}{-2} = \frac{y-1}{3} = \frac{z-1}{-1}$  passing through  $(0, 1, 1)$  and having d.r.  $\langle -2, 3, -1 \rangle$

Since the plane contains the line and the point

$$\Rightarrow a(1) + b(1) + c(1 - 2) = 0 \Rightarrow a + b - c = 0 \dots\dots(2)$$

Also the line and normal to the plane are perpendicular

$$\Rightarrow -2a + 3b - c = 0 \dots\dots\dots(3)$$

Solving (2) and (3)  $\frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k$

Hence required equation of the plane is

$$\Rightarrow 2(x + 1) + 3(y - 0) + 5(z - 2) = 0$$

$$\Rightarrow 2x + 3y + 5z - 8 = 0$$

**Method -II**

Let the equation of the plane be  $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$  is passing through  $(-1, 0, 2)$

$$\Rightarrow a(x + 1) + b(y - 0) + c(z - 2) = 0 \dots\dots\dots(1)$$

Given line  $\frac{x-0}{-2} = \frac{y-1}{3} = \frac{z-1}{-1}$  passing through  $(0, 1, 1)$  and having d.r.  $\langle -2, 3, -1 \rangle$

Since the plane contains the line and the point

$$\Rightarrow a(1) + b(1) + c(1 - 2) = 0 \Rightarrow a + b - c = 0 \dots\dots(2)$$

Also the line and normal to the plane are perpendicular

$$\Rightarrow -2a + 3b - c = 0 \dots\dots\dots(3)$$

Hence required equation of the plane is

$$\begin{vmatrix} x + 1 & y & z - 2 \\ 1 & 1 & -1 \\ -2 & 3 & -1 \end{vmatrix} = 0$$

$$\Rightarrow 2(x + 1) + 3(y - 0) + 5(z - 2) = 0$$

$$\Rightarrow 2x + 3y + 5z - 8 = 0$$

**Question 18**

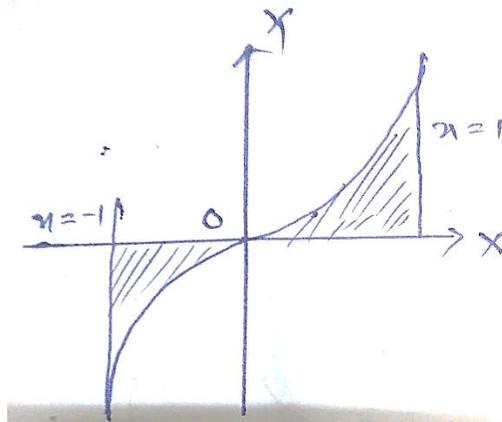
[4]

(i) Given, the equation of the curve:  $y = x|x|$

$$\therefore y = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$$

Hence sketch of the region enclosed bounded by the curve,  $y = x|x|$  and the ordinates  $x = -1$  and  $x = 1$  is as follows.

(ii)



(iii)  $\int_0^1 x^2 dx = \left[ \frac{x^3}{3} \right]_0^1 = \frac{1}{3}$

$\therefore$  Area of the shaded region  $= 2 \int_0^1 x^2 dx = \frac{2}{3}$  sq. units.

### SECTION C - 15 MARKS

#### Question 19

[5]

(i) (b) MC = AC

(ii) (d) III and IV are correct

(iii) Given:  $\bar{x} = 53$ ,  $\bar{y} = 28$ ,  $b_{yx} = -1.2$ ,  $b_{xy} = -0.3$

Using,  $r^2 = b_{yx} \times b_{xy}$   
 $= -1.2 \times -0.3 = 0.36$

$\Rightarrow r = -0.6$

**ANS : - 0.6**

(iv) Given,  $R(x) = 36x + 3x^2 + 5$

$\Rightarrow MR = \frac{d}{dx}(36x + 3x^2 + 5)$   
 $= 36 + 6x$

$\therefore MR(x = 5) = 66$

**ANS : 66**

- (v) Given  $C(x) = 210x + 7000$ ,  $R(x) = 280x$   
 Minimum number must be sold daily when  $R(x) = C(x)$   
 $\Rightarrow 210x + 7000 = 280x$   
 $\Rightarrow 70x = 7000$   
 $\Rightarrow x = 100$ .  
 $\therefore$  Minimum number that must be sold is 100.  
**ANS: 100**

**Question 20**

[2]

(i)  $C(x) = 200 + 20x - \frac{1}{2}x^2$   
 $\Rightarrow AC = \frac{200}{x} + 20 - \frac{1}{2}x$

To find AC falls:

$$\frac{d}{dx}(AC) = \frac{d}{dx}\left(\frac{200}{x} + 20 - \frac{1}{2}x\right)$$

$$= -\frac{200}{x^2} - \frac{1}{2} = -\left(\frac{200}{x^2} + \frac{1}{2}\right) < 0, \forall x > 0.$$

Hence AC falls continuously.

**OR**

(ii) Given,  $x = 100 - 4p$   
 $\Rightarrow p = \frac{100 - x}{4}$   
 $\therefore R(x) = px = \frac{100x - x^2}{4}$   
 $\Rightarrow MR = \frac{d}{dx}\left(\frac{100x - x^2}{4}\right) = \frac{100 - 2x}{4}$   
 $\therefore MR = 0$   
 $\therefore x = 50$

**Question 21**

[4]

(i)  $\bar{x} = \frac{\sum x}{n} = 170$ ,  $\bar{y} = \frac{\sum y}{n} = 192$ ,  
 $b_{yx} = r \frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{20}{60} = 0.2$   
 Regression equation y on x is  
 $y - 192 = 0.2(x - 170)$

$$\Rightarrow y - 192 = 0.2x - 34$$

$$\Rightarrow y = 0.2x + 158$$

Put  $x = 200$

$$\Rightarrow y = 0.2(200) + 158 = 198$$

$\therefore$  Expenditure on food and entertainment = ₹ 198.

**OR**

(ii)

(a) Regression lines are  $3x + 2y - 26 = 0$  .....(i)  
 $6x + y - 31 = 0$  ..... (ii)

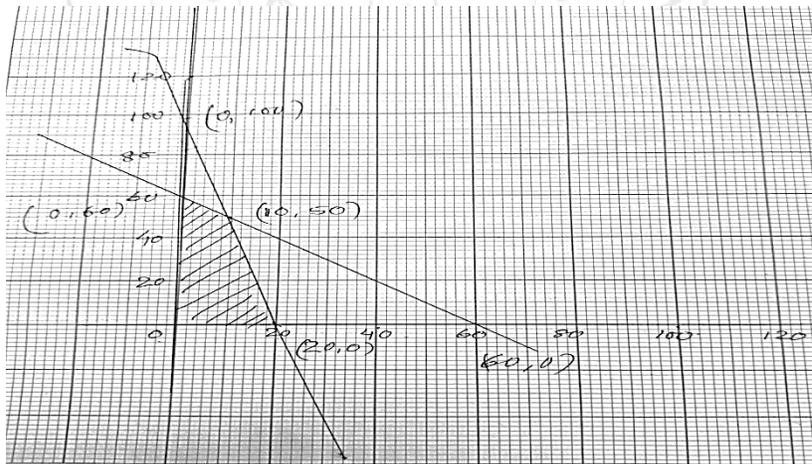
$(\bar{x}, \bar{y})$  satisfies the equation (i) and (ii),  
 $\therefore$  Solving (i) and (ii) we get,  $\bar{x} = 4$  and  $\bar{y} = 7$ .

(b) On finding we get,  $b_{yx} = -\frac{1}{6}$ ,  $b_{xy} = -\frac{3}{2}$   
 $\therefore r^2 = b_{yx} \times b_{xy} = -\frac{1}{6} \times -\frac{3}{2} = \frac{1}{4}$   
 $\therefore r = -\frac{1}{2}$  [ $\because b_{yx}, b_{xy}$  and  $r$  must have same sign]

**Question 22**

**[4]**

(i) From graph, corner points are: A (0, 60), B (10, 50), C (20, 0), D (0, 0)



(ii)  $Z = px + qy$   
 Given,  $Z$  is maximum at (0, 60) and (10, 50)  
 $\therefore 0 \cdot p + 60 \cdot q = 10 \cdot p + 50 \cdot q$   
 $\Rightarrow 10p = 10q$   
 $\Rightarrow p = q$   
 So there can be infinite number of optimal solutions.