Strictly Confidential — (For Internal and Restricted Use Only)

Senior Secondary School Term II Compartment Examination, 2022

Marking Shceme – Mathematics (041)

(Paper Code – 65/6/3)

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. "Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-X, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark (✓) wherever answer is correct. For wrong answer 'X" be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks __________(example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-

- Leaving answer or part thereof unassessed in an answer book.
- Giving more marks for an answer than assigned to it.
- Wrong totaling of marks awarded on a reply.
- Wrong transfer of marks from the inside pages of the answer book to the title page.
- Wrong question wise totaling on the title page.
- Wrong totaling of marks of the two columns on the title page.
- Wrong grand total.
- Marks in words and figures not tallying.
- Wrong transfer of marks from the answer book to online award list.
- Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
- Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 65/6/3
EXPECTED ANSWER/VALUE POINTS
SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a}, \vec{b} + \vec{c} = \vec{0}$, then find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

Ans. $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$\vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{a} + \vec{b} + \vec{c})(\vec{a} + \vec{b} + \vec{c}) = 0$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ 1

$\Rightarrow 3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0$ $\frac{1}{2}$

$\Rightarrow \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-3}{2}$ $\frac{1}{2}$

2. (a) Find the general solution of the differential equation x cos y dy = (x log x + 1) e^x dx.

OR

(b) Find the value of (2a – 3b), if a and b represent respectively the order the degree of the differential equation $x\left[y\left(\frac{d^2y}{dx^2}\right)^3 + x\left(\frac{dy}{dx}\right)^2 - \frac{y}{x}\frac{dy}{dx}\right] = 0$.

Ans. $x \cos y \, dy = (x \log x + 1)e^x \, dx$

$\Rightarrow \int \cos y \, dy = \int \left(\log x + \frac{1}{x}\right) \cdot e^x \, dx$ $\frac{1}{2}$

$\Rightarrow \sin y = \log x.e^x + C$ $\left(\because \int [f(x) + f'(x)] \cdot e^x dx = f(x) \cdot e^x + C\right)$ $1\frac{1}{2}$

OR

order = 2, degree = 3 $1+\frac{1}{2}$

$\therefore$ 2a – 3b = 4 – 9 = –5 $\frac{1}{2}$

3. Evaluate:

$$\int_1^2 \log\left(\frac{3}{x}-1\right)dx$$

Ans. $I = \int_1^2 \log\left(\frac{3}{x}-1\right)dx$

$= \int_1^2 [\log(3-x) - \log x]\,dx$ $\frac{1}{2}$

$= \int_1^2 \log(3-x)\,dx - \int_1^2 \log x\,dx$

$= \int_1^2 \log(3-x)\,dx - \int_1^2 \log(3-x)\,dx$ $(\therefore$ Using $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ 1

$= 0$ $\frac{1}{2}$

4. If $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - \hat{k}$ are three vectors, then find a vector perpendicular to both the vectors $(\vec{a}+\vec{b})$ and $(\vec{b}-\vec{c})$.

Ans. $\vec{a} + \vec{b} = 3\hat{j}$, $\vec{b} - \vec{c} = 3\hat{k}$ $\frac{1}{2}+\frac{1}{2}$

Vector perpendicular to $\vec{a}+\vec{b}$ and $\vec{b}-\vec{c}$

$= (3\hat{j}) \times (3\hat{k}) = 9\hat{i}$ 1

5. One bag contains 4 white and 5 black balls. Another bag contains 6 white and 7 black balls. A ball, drawn at random, is transferred from the first bag to the second bag and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

Ans. **Case I:** White ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{4}{9} \times \frac{7}{14}$ $\frac{1}{2}$

Case II: Black ball is transferred from bag I to bag II

P(white ball from bag II) = $\frac{5}{9} \times \frac{6}{14}$ $\frac{1}{2}$

Total Probability = $\frac{4}{9}\times\frac{7}{14}+\frac{5}{9}\times\frac{6}{14}$ $\frac{1}{2}$

$= \frac{29}{63}$ $\frac{1}{2}$

6. A bag contains cards numbered 1 to 25. Two cards are drawn at random, one after the other, without replacement. Find the probability that the number on each card is a multiple of 7.

Ans. Multiples of 7 from 1 to 25 are 7, 14, 21 $\frac{1}{2}$

P (number on each card is a multiple of 7)

$= \frac{3}{25}\times\frac{2}{24}=\frac{1}{100}$ $1\frac{1}{2}$

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. Find the distance between the point (3, 4, 5) and the point where the line $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}$ meets the plane $x + y + z = 17$.

Ans. Any point on the given line is $(\lambda + 3, 2\lambda + 4, 2\lambda + 5)$. 1

If this point is the point of intersection of the line and plane,

then $(\lambda + 3) + (2\lambda + 4) + (2\lambda + 5) = 17 \Rightarrow \lambda = 1$ $\frac{1}{2}$

$\therefore$ Point of intersection is (4, 6, 7) $\frac{1}{2}$

Required distance = $\sqrt{(4-3)^2+(6-4)^2+(7-5)^2} = 3$ 1

8. (a) Find the distance between the following parallel lines:

$$\vec{r} = (2\hat{i}+\hat{j}-\hat{k})+\lambda(\hat{i}+\hat{j}-\hat{k})$$

$$\vec{r} = (\hat{i}+2\hat{j}-\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$$

OR

(b) Find the coordinates of the point where the line through the points (–1, 1, –8) and (5, –2, 10) crosses the ZX-plane.

Ans. (a) $\vec{a}_2 - \vec{a}_1 = -\hat{i} - 3\hat{j} + 2\hat{k}, \ \vec{b} = \hat{i} + \hat{j} - \hat{k}$ $\frac{1}{2}$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -3 & 2 \\ 1 & 1 & -1 \end{vmatrix} = \hat{i} + \hat{j} + 2\hat{k}$$ 1

Required distance = $\frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$

$= \frac{\sqrt{1+1+4}}{\sqrt{1+1+1}} = \sqrt{\frac{6}{3}} = \sqrt{2}$ $1\frac{1}{2}$

OR

(b) Equation of line through (–1, 1, –8) and (5, –2, 10)

is $\frac{x+1}{5-(-1)} = \frac{y-1}{-2-1} = \frac{z+8}{10-(-8)}$

i.e. $\frac{x+1}{6} = \frac{y-1}{-3} = \frac{z+8}{18} = \lambda$ 1

Any point on this line is $(6\lambda - 1, -3\lambda + 1, 18\lambda - 8)$ $\frac{1}{2}$

Line crosses ZX-plane i.e. $y = 0$

$\Rightarrow -3\lambda + 1 = 0 \Rightarrow \lambda = \frac{1}{3}$ 1

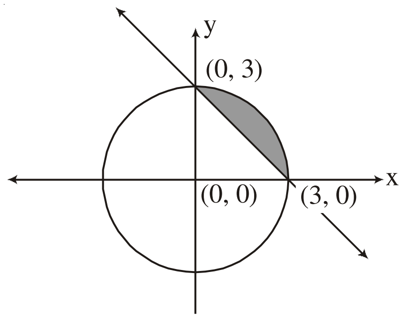
Required point is (1, 0, –2) $\frac{1}{2}$

9. (a) Find the area of the region $\{(x, y) : x^2 + y^2 \leq 9, x + y \geq 3\}$, using integration.

OR

(b) Using integration, find the area of the region bounded by the parabola $y^2 = 4x$, the lines $x = 0$ and $x = 3$ and the x-axis.

Ans. Point of intersection (3, 0) and (0, 3)



Correct figure $\frac{1}{2}$

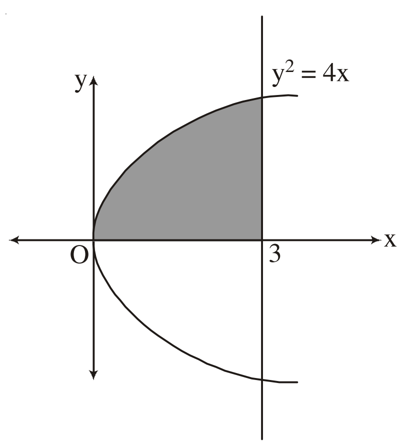
Required Area

$$= \int_0^3 \sqrt{9-x^2}\,dx - \int_0^3 (3-x)dx \qquad 1$$

$$= \left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3 - \left[\frac{(3-x)^2}{-2}\right]_0^3 \qquad 1$$

$$= \frac{9}{2}\sin^{-1}1 - \frac{9}{2} = \frac{9}{2}\left(\frac{\pi}{2}-1\right) \qquad \frac{1}{2}$$

OR



Correct Figure 1

Required Area = $\int_0^3 2\sqrt{x}\, dx$ 1

$= 2\times\frac{2}{3}[x^{3/2}]_0^3$ $\frac{1}{2}$

$= \frac{4}{3}\times 3^{3/2} = 4\sqrt{3}$ $\frac{1}{2}$

10. Find:

$\int \sin 2x \sin 3x\, dx$

Ans. I = $\int \sin 2x \sin 3x\, dx$

$= \frac{1}{2}\int 2\sin 2x \sin 3x\, dx$ 1

$= \frac{1}{2}\int (\cos x - \cos 5x)\, dx$ 1

$= \frac{1}{2}\left[\sin x - \frac{\sin 5x}{5}\right] + C$ 1

SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. (a) Find:

$\int \cos x \bullet \tan^{-1}(\sin x)\, dx$

OR

(b) Find:

$\int \frac{e^x}{(e^x+1)(e^x+3)}\, dx$

Ans. I = $\int \cos x \cdot \tan^{-1}(\sin x)\, dx$

Put $\sin x = t \Rightarrow \cos x\, dx = dt$ 1

$\therefore\ I = \int \tan^{-1} t \cdot 1\, dt$

$$= \tan^{-1} t \cdot t - \frac{1}{2}\int \frac{2t}{1+t^2}\, dt$$ $1\frac{1}{2}$

$$= t \cdot \tan^{-1} t - \frac{1}{2}\log|1+t^2| + C$$ 1

$$= \sin x \cdot \tan^{-1}(\sin x) - \frac{1}{2}\log|1+\sin^2 x| + C$$ $\frac{1}{2}$

OR

$$I = \int \frac{e^x}{(e^x+1)(e^x+3)}\, dx$$

Put $e^x = t \Rightarrow e^x\, dx = dt$ 1

$$I = \int \frac{dt}{(t+1)\,(t+3)}$$

$$= \frac{1}{2}\int \left(\frac{1}{t+1} - \frac{1}{t+3}\right) dt$$ $1\frac{1}{2}$

$$= \frac{1}{2}[\log|t+1| - \log|t+3|] + C$$ 1

$$= \frac{1}{2}[\log|e^x+1| - \log|e^x+3|] + C$$ $\frac{1}{2}$

$$\text{or } \frac{1}{2}\log\left|\frac{e^x+1}{e^x+3}\right| + C$$

12. Find the particular solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \tan x$, given $y(0) = 1$.

Ans. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{2x}{1+x^2} \cdot y = \frac{\tan x}{1+x^2}$$ $\frac{1}{2}$

$$\text{I.F.} = e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = 1+x^2$$ 1

Solution is given by

$$y \cdot (1 + x^2) = \int \frac{\tan x}{1+x^2} \cdot (1+x^2)\, dx$$ 1

$= \int \tan x \, dx$

$= \log|\sec x| + C$ $\frac{1}{2}$

When x = 0, y = 1 gives C = 1 $\frac{1}{2}$

Required particular solution is $y \cdot (1 + x^2) = \log|\sec x| + 1$ $\frac{1}{2}$

or $y = \frac{\log|\sec x|}{1+x^2} + \frac{1}{1+x^2}$

13. Find the equation of the plane passing through the intersection of the planes $\vec{r} \cdot (2\hat{i}+2\hat{j}-3\hat{k}) = 7$ and $\vec{r} \cdot (2\hat{i}+5\hat{j}-3\hat{k}) = 9$ and through the point (2, 1, 3).

Ans. Equation of required plane is

$\vec{r} \cdot [(2\hat{i}+2\hat{j}-3\hat{k}) + \lambda(2\hat{i}+5\hat{j}+3\hat{k})] = 7 + 9\lambda$ 1

or $\vec{r} \cdot [(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k})] = 7 + 9\lambda$

As the plane passes through (2, 1, 3), we have

$(2\hat{i}+\hat{j}+3\hat{k}) \cdot [(2+2\lambda)\hat{i} + (2+5\lambda)\hat{j} + (-3+3\lambda)\hat{k}] = 7 + 9\lambda$ 1

$\Rightarrow 2(2 + 2\lambda) + 1(2 + 5\lambda) + 3(-3 + 3\lambda) = 7 + 9\lambda$

$\Rightarrow 9\lambda = 10 \Rightarrow \lambda = \frac{10}{9}$ 1

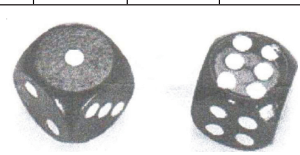
Required plane is $\vec{r} \cdot \left(\frac{38}{9}\hat{i} + \frac{68}{9}\hat{j} + \frac{3}{9}\hat{k}\right) = 17$ 1

or $\vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$

Case-Study Based Question

14. A biased die is tossed and respective probabilities for various faces to turn up are the following:

Face	1	2	3	4	5	6
Probability	0.1	0.24	0.19	0.18	0.15	K



Based on the above information, answer the following questions:

(a) What is the value of K?

(b) If a face showing an even number has turned up, then what is the probability that it is the face with 2 or 4?

Ans. (a) $P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1$ 1

$\Rightarrow$ $0.1 + 0.24 + 0.19 + 0.18 + 0.15 + K = 1$

$\Rightarrow$ $K = 0.14$ 1

(b) A: face shows 2 or 4

B: even face have turned up

Required probability = $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

here $P(A \cap B) = P(2) + P(4)$

$= 0.24 + 0.18 = 0.42$ 1

$P(B) = P(2) + P(4) + P(6) = 0.56$ $\frac{1}{2}$

$\therefore$ $P(A/B) = \dfrac{0.42}{0.56} = \dfrac{3}{4}$ $\frac{1}{2}$