Secondary School Term II Compartment Examination, 2022

Marking Scheme — Mathematics 30/6/3
Subject Code – 041

General Instructions:

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. "Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-X, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark($\sqrt{}$) wherever answer is correct. For wrong answer 'X" be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. This is most common mistake which evaluators are committing.

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks _____(example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-

    • Leaving answer or part thereof unassessed in an answer book.

    • Giving more marks for an answer than assigned to it.

    • Wrong totaling of marks awarded on a reply.

    • Wrong transfer of marks from the inside pages of the answer book to the title page.

    • Wrong question wise totaling on the title page.

    • Wrong totaling of marks of the two columns on the title page.

    • Wrong grand total.

    • Marks in words and figures not tallying.

    • Wrong transfer of marks from the answer book to online award list.

    • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)

    • Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

QUESTION PAPER CODE 30/6/3
EXPECTED ANSWER/VALUE POINTS
SECTION A

Question numbers 1 to 6 carry 2 marks each.

1. If the pth term of an A.P. is $\dfrac{1}{q}$ and the qth term is $\dfrac{1}{p}$, then show that $(pq)^{th}$ term is 1.

Ans. $a_p = a + (p - 1)\, d = \dfrac{1}{q}$ $\qquad$ ...(i)

$a_q = a + (q - 1)\, d = \dfrac{1}{p}$ $\qquad$ ...(ii) $\dfrac{1}{2}$

Solving (i) and (ii)

$d = \dfrac{1}{pq},\ a = \dfrac{1}{pq}$ $\dfrac{1}{2}+\dfrac{1}{2}$
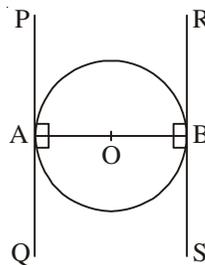
$a_{pq} = a + (pq - 1)\, d$

$= \dfrac{1}{pq} + (pq - 1)\dfrac{1}{pq}$

$= 1$ $\dfrac{1}{2}$

2. Prove that the tangents drawn at the end points of the diameter of a circle are parallel.

Ans.

Correct Figure $\dfrac{1}{2}$

$\angle OAP = \angle OAQ = \angle OBR = \angle OBS = 90°$ $\dfrac{1}{2}$

$\Rightarrow \angle OAP = \angle OBS$ and $\angle OAQ = \angle OBR$ $\dfrac{1}{2}$

But they are alternate interior angles

$\therefore$ PQ ‖ RS $\dfrac{1}{2}$

3. In the following cumulative frequency table, find the values of a, b, c and d.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	7	a	5	b
Cumulative Frequency	5	c	18	d	30

Ans. Here c = 12, a = 6, d = 23, b = 7 $\frac{1}{2}$×4=2

4. (a) Find the number of terms of the A.P.?

   293, 285, 277, ..., 53

<div align="center">OR</div>

   (b) Find the sum of the first 40 positive intergers divisible by 7.

Ans. (a) Here a = 293, d = –8 $\frac{1}{2}$

   t_n = 293 + (n – 1) (–8) = 53 $\frac{1}{2}$

   293 –8n + 8 = 53

   8n = 248

   $\Rightarrow$ n = 31 1

   $\therefore$ Number of terms = 31

<div align="center">OR</div>

   (b) 7, 14, 21, ..., 40 terms $\frac{1}{2}$

   $S_{40} = \frac{40}{2}\{14 + 39 \times 7\}$ 1

   = 20 (14 + 273)

   = 20 × 287

   = 5740 $\frac{1}{2}$

5. (a) Solve for x:

$$2x^2 - 2\sqrt{2}x + 1 = 0$$

OR

(b) Find the value (s) of k for which the quadratic equation $x^2 + 5kx + 16 = 0$ has real and equal roots.

Ans. (a) $2x^2 - 2\sqrt{2}x + 1 = 0$

$$x = \frac{2\sqrt{2} \pm \sqrt{8 - 8}}{4}$$ 1

$$\Rightarrow x = \frac{1}{\sqrt{2}}, \ \frac{1}{\sqrt{2}}$$ 1

OR

(b) For real and equal roots, Disc = 0

$$\therefore 25k^2 - 64 = 0$$ 1

$$\Rightarrow k^2 = \frac{64}{25}$$

$$\Rightarrow k = \pm\frac{8}{5}$$ 1

6. If the mean of the following frequency distribution is 10.8, then find the value of p:

Class	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	p	5	8	2

Ans.

Class Internal	f_i	x_i	$f_i x_i$
0 – 4	3	2	6
4 – 8	p	6	6p
8 – 12	5	10	50
12 – 16	8	14	112
16 – 20	2	18	36
	18 + p		204 + 6p

Correct Table   1

$$10.8 = \frac{204 + 6p}{18 + p}$$ $\frac{1}{2}$

$$p = 2$$ $\frac{1}{2}$

SECTION B

Question numbers 7 to 10 carry 3 marks each.

7. (a) The radius of the base and the height of a solid right circular cylinder are in the ration 2 : 3 and its volume is 1617 cm³. Find the total surface area of the cylinder. $(\text{Take}\,\pi = \dfrac{22}{7})$

OR

(b) A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller solid cones, each of radius 3.5 cm and height 3 cm. Find the number of cones os formed.

Ans. (a) Let the height of right cylinder = h

$\text{Base} = \dfrac{2}{3}\,h$ $\dfrac{1}{2}$

$\therefore \text{Volume} = \pi\left(\dfrac{2}{3}h\right)^2 \times h = 1617$ $\dfrac{1}{2}$

$\Rightarrow h^3 = \dfrac{7^3 \times 3^3}{2^3}$

$\Rightarrow h = \dfrac{21}{2} = 10.5$ 1

Total surface area = 2πr $(l + r)$

$= 2 \times \dfrac{22}{7} \times 7\,(17.5)$ $\dfrac{1}{2}$

$= 770 \text{ cm}^2$ $\dfrac{1}{2}$

OR

(b) Volume of solid sphere $= \dfrac{4}{3}\pi\,(10.5)^3$ $\dfrac{1}{2}$

Volume of Cone $= \dfrac{1}{3}\pi\,(3.5)^2.\,3$ $\dfrac{1}{2}$

Let the number of cones be n

$$\therefore \; n \times \frac{1}{3}\pi \; (3.5)^2. \; 3 \; = \; \frac{4}{3}\pi \; (10.5)^3 \qquad\qquad 1$$

$$n \; = \; \frac{4}{3} \times \frac{(10.5)^3}{(3.5)^2}$$

$$= 126 \qquad\qquad\qquad 1$$

$$\therefore \text{ No. of Cones formed} = 126$$

8. A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.

Ans. h = 20 m, r = $\frac{7}{2}$

Volume of earth dug out = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20$$

$$= \left(\frac{22 \times 7 \times 7 \times 20}{7 \times 2 \times 7} \right) m^3 \qquad\qquad 1$$

Area of base of platform = $(22 \times 14) \; m^2$ $\qquad\qquad \frac{1}{2}$

Height of platform = $\dfrac{22 \times 7 \times 7 \times 20}{7 \times 2 \times 2 \times 22 \times 14}$ $\qquad\qquad 1$

$$= \frac{5}{2} = 2.5 \; m \qquad\qquad\qquad \frac{1}{2}$$
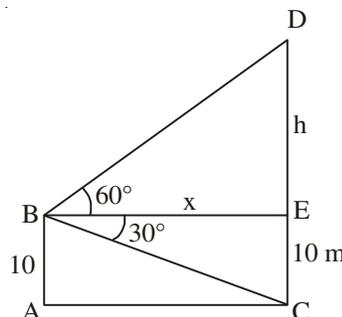
9. Construct a tangent to a circle of radius 3 cm from a point on the concentric circle of radius 6 cm.

Ans. Correct and accurate construction. $\qquad\qquad\qquad 3$

10. A man standing on the deck of a ship, which is 10 m above the water level, observes that the angle of elevation of the top of a hill is 60° and the angle of depression of the base of the hill is 30°. Find the height of the hill.

Ans. Let DE = h and AC = BE = x $\qquad\qquad\qquad$ Correct Figure $\quad$ 1

In $\triangle$BDE, $\dfrac{h}{x}$ = tan 60° $\qquad\qquad\qquad \frac{1}{2}$

$\Rightarrow \dfrac{h}{x} = \sqrt{3}$

$\Rightarrow h = \sqrt{3}x$

In ΔBEC, $\dfrac{10}{x} = \tan 30° = \dfrac{1}{\sqrt{3}}$ $\dfrac{1}{2}$

$\Rightarrow x = 10\sqrt{3}$

$\therefore h = \sqrt{3} \times 10\sqrt{3} = 30$ m $\dfrac{1}{2}$

$\therefore$ Height of hill $= (30 + 10)$ m $= 40$ m. $\dfrac{1}{2}$

SECTION C

Question numbers 11 to 14 carry 4 marks each.

11. (a) In Figure 1, two circle touch externally at P. A common tangent touches them at A and B and another common tangent is at P, which meets the common tangent AB at C. Prove that $\angle APB = 90°$.
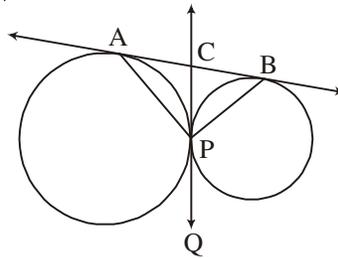

Figure 1

OR

(b) In Figure 2, PQ and LM are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting PQ at A and LM at B. Prove that $\angle AOB = 90°$.
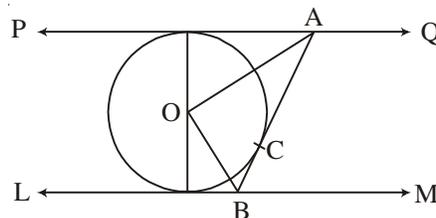

Figure 2

Ans. (a) Here CA = CP

$\Rightarrow \angle 1 = \angle 2$ ...(i)   1
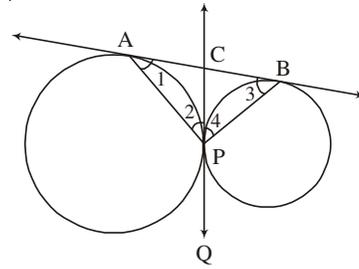
Again CB = CP

$\Rightarrow \angle 3 = \angle 4$ ...(ii)   1

Adding (i) and (ii), we have

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ (angles of ΔAPB)   1

$\therefore \angle APB = 90°$ 1

OR

(b) $\Delta ADO \cong \Delta ACO$

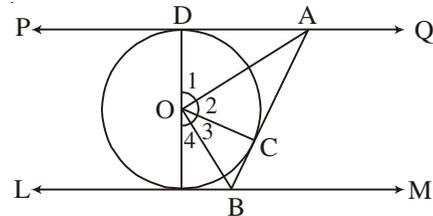$\Rightarrow \angle 1 = \angle 2$ ...(i)   1

Similarly $\angle 3 = \angle 4$ ...(ii)   1

$\therefore \angle 1 + \angle 4 = \angle 2 + \angle 3$ (using (i) and (ii))

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180°$ 1

$\Rightarrow \angle 2 + \angle 3 = 90°$

i.e. $\angle AOB = 90°$ 1

12. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of Q is 45°. Find the height of the tower PQ and the distance XP. (Use $\sqrt{3}$ = 1.732)
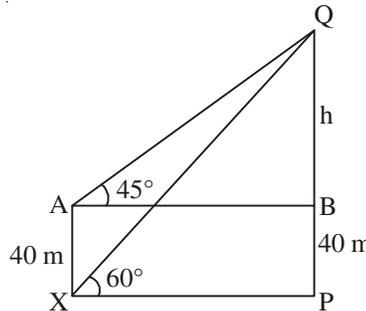
Ans. Let QB = h, AB = XP = x   Correct Figure   1

$\dfrac{BQ}{AB} = \tan 45°$

h = x   ...(i) $\dfrac{1}{2}$

$\dfrac{40 + h}{x} = \tan 60° = \sqrt{3}$ $\dfrac{1}{2}$

$\Rightarrow 40 + h = \sqrt{3}h$

$\Rightarrow 40 = h\left(\sqrt{3} - 1\right)$

$\Rightarrow h = \dfrac{40}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = 20\left(\sqrt{3}+1\right)$ 1

$= 20\,(1.732 + 1) = 54.64$

$\therefore$ PQ = h + 40 = 40 + 54.64 = 94.64 m. $\dfrac{1}{2}$

XP = 54.64 m. $\dfrac{1}{2}$

Height of the Tower = 94.64 m.

XP = 54.64 m

Case Study – 1

13. Health insurace is an agreement whereby the insurance company agrees to undertake a guarantee of compensation for medical expenses in case the insured falls ill or meets with an accident which leads to hospitalisation of the insured. The government also promotes health insurance by providing a deduction from income tax.

An SBI health insurance agent found the following data for distribution of ages of 100 policy holders. The health insurance policies are given to persons having age 15 years and onwards but less than 60 years.

Age (in years)	Number of Policy Holders
15 – 20	2
20 – 25	4
25 – 30	18
30 – 35	21
35 – 40	33
40 – 45	11
45 – 50	3
50 – 55	6
55 – 60	2

(a) Find the modal age of the policy holders.

(b) Find the median age of the policy holders.

Ans.

Age	Frequency	Cumulative Frequency
15 – 20	2	2
20 – 25	4	6
25 – 30	18	24
30 – 35	21	45
35 – 40	33	78
40 – 45	11	89
45 – 50	3	92
50 – 55	6	98
55 – 60	2	100

(a)   Here Max. frequency = 33, $\therefore$ l = 35

f_1 = 33, f_0 = 21, f_2 = 11, h = 5 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{1}{2}$

$\therefore$ Mode = $1 + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) h$

$= 35 + \left(\dfrac{33 - 21}{66 - 21 - 11} \right) \times 5 \qquad\qquad\qquad\qquad\qquad\qquad 1$

$= 36.8 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{1}{2}$

(b)   N = 100, $\dfrac{N}{2}$ = 50

$\therefore$ Median class is 35 – 40

Here l = 35, f = 33, C = 45 $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \frac{1}{2}$

$\therefore$ Median = $l + \left(\dfrac{\dfrac{N}{2} - C}{f} \right) \times h$

$$= 35 + \frac{50-45}{33} \times 5$$

1

$$= 35 + 0.76 = 35.76$$

$\frac{1}{2}$

Here Modal age of the policy holders = 36.8 years.

and Median age of the policy holders = 35.76 years.

Case Study – 2

14. Social work aims at fulfilment of human needs. Social workers aim to open the doors of access and opportunity for those who are in greatest need. Free education is a great social work. By doint so, we can remove illiteracy from our society.

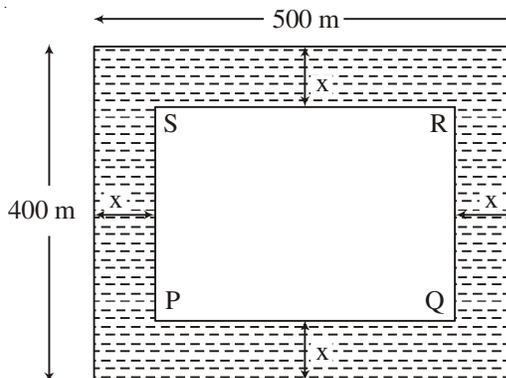Rohan, being a social worker, wants to donate his land to the Village Panchayat for opening of a school.

Figure 4

Rohan's land is in the form of a rectangle of dimensions 500 m × 400 m. The Village Panchayat decides to leave the area on all the four sides of the land for grass and flowers. If width fo x m land is kept for grass fand flowers on all the four sides (as shown in Figure 4);

(a) Find the lengths PQ and QR if area of grass and flowers region surrounding PQRS is 118400 m².

(b) Also, find the perimeter of the rectangle PQRS.

Ans. (a) Area of rectangle PQRS = 118400

∴ (500 – 2x) (400 – 2x) = 118400

1

⇒ x^2 – 450x + 29600 = 0

⇒ x = 370 or x = 80

$\frac{1}{2}$

x = 370 is not possible,

∴ x = 80

∴ PQ = 500 – 160 – 340 m.

QR = 400 – 160 = 240 m. $\frac{1}{2}$

(b) Perimeter = 2(340 + 240) = 1160 m.   2