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**Senior Secondary School Term II Examination, 2022**

**Marking Scheme – MATHEMATICS (SUBJECT CODE – 041)**

**(PAPER CODE – 65/5/1 )**

**General Instructions: -**

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."**

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency-based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark( √ ) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totalled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks __________(example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totalling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totalling on the title page.
    - Wrong totalling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

14. Any unassessed portion, non-carrying over of marks to the title page, or totalling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totalled and written in figures and words.

17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

**MARKING SCHEME**

Senior Secondary School Examination TERM–II, 2022

**MATHEMATICS (Subject Code–041)**

**[ Paper Code : 65/5/1 ]**

**Maximum Marks : 40**

| Q. No. | EXPECTED ANSWER / VALUE POINTS | Marks |
|---|---|---|
| | **SECTION—A**<br>**Question Nos. 1 to 6 carry 2 marks each.** | |
| **Q1.** | Find : $\int \frac{dx}{\sqrt{4x-x^2}}$ | **2** |
| **A1.** | $\int \frac{dx}{\sqrt{4x-x^2}}$<br>$= \int \frac{dx}{\sqrt{2^2-(x-2)^2}}$<br>$= \sin^{-1}\left(\frac{x-2}{2}\right) + C$ | <br>1<br>1 |
| **Q2.** | Find the general solution of the following differential equation :<br>$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ | **2** |
| **A2.** | $\int \frac{dy}{e^{-y}} = \int (x^2 + e^x)\, dx$<br>$e^y = \frac{x^3}{3} + e^x + C$ | 1<br>1 |
| **Q3.** | Let X be a random variable which assumes values $x_1, x_2, x_3, x_4$ such that $2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4)$.<br>Find the probability distribution of X. | **2** |
| **A3.** | Let $P(X = x_3) = k$<br>$P(X = x_1) = \frac{k}{2},\ P(X = x_2) = \frac{k}{3},\ P(X = x_4) = \frac{k}{5}$<br>$\frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$<br>$k = \frac{30}{61}$ | ½<br><br><br>½ |

Probability Distribution of X is:

| $X$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|---|---|---|---|---|
| $P(X)$ | 15/61 | 10/61 | 30/61 | 6/61 |

1

**Q4.** If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{a}.\vec{b} = 1$ and $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $|\vec{b}|$ — **2**

**A4.** Let $\vec{b} = x\hat{i} + y\hat{i} + z\hat{k}$

$$\vec{a}\cdot\vec{b} = 1 \Rightarrow x + y + z = 1 \quad \ldots (1)$$ ½

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{i}\,(z-y) - \hat{j}\,(z-x) + \hat{k}(y-x) = \hat{j} - \hat{k}$$ ½

$$z - y = 0 \Rightarrow y = z$$

$$\Rightarrow x - z = 1 \quad \ldots (2)$$

$$x - y = 1 \quad \ldots (3)$$

Solving (1), (2), (3)

$$x = 1, \quad y = 0, \quad z = 0$$ ½

$$\vec{b} = \hat{i}\,, \text{ so } |\vec{b}| = 1$$ ½

**Q5.** If a line makes an angle $\alpha, \beta, \gamma$ with the coordinate axes, then find the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$. — **2**

**A5.** $l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ — 1

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma$

$$= 2\cos^2\alpha - 1 + 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$$

$$= 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3$$

$$= 2 - 3$$

$$= -1$$ 1

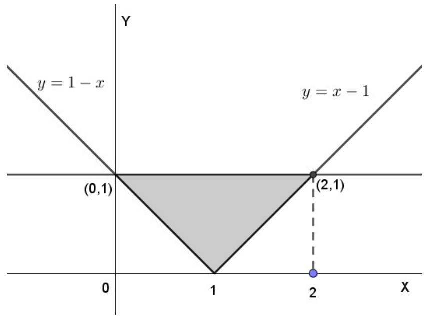
| | | |
|---|---|---|
| **Q6.** | (a) Events A and B are such that<br>$P(A)=\frac{1}{2}$, $P(B)=\frac{7}{12}$ and $P(\bar{A}\cup\bar{B})=\frac{1}{4}$<br>Find whether the events A and B are independent or not.<br>**OR**<br>(b) A box $B_1$ contains 1 white ball and 3 red balls. Another box $B_2$ contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes $B_1$ and $B_2$, then find the probability that the two balls drawn are of the same colour. | **2** |
| **A6(a).** | $P(A)=\frac{1}{2},\ P(B)=\frac{7}{12},\ P(\bar{A}\cup\bar{B})=\frac{1}{4}$<br>$P(\bar{A}\cup\bar{B})=P(\overline{A\cap B})=1-P(A\cap B)$<br>$P(A\cap B)=1-\frac{1}{4}=\frac{3}{4}$<br>$P(A)\times P(B)=\frac{1}{2}\times\frac{7}{12}=\frac{7}{24}$<br>$P(A)\times P(B)\neq P(A\cap B)$<br>$\therefore$ $A$ and $B$ are not independent | ½<br>½<br>½<br><br>½ |
| **A6.(b)** | **Or**<br>$P$ (both balls drawn are of same colour)<br>= $P$ (both white) + $P$ (both red)<br>$=\frac{1}{4}\times\frac{2}{5}+\frac{3}{4}\times\frac{3}{5}=\frac{11}{20}$ | 1<br>1 |
| | **SECTION—B**<br>**Question Nos. 7 to 10 carry *3* marks each.** | |
| **Q7.** | Evaluate :<br>$\int_0^{\pi/4}\frac{dx}{1+\tan x}$ | **3** |
| **A7.** | $I=\int_0^{\pi/4}\frac{dx}{1+\tan x}$<br>$I=\int_0^{\pi/4}\frac{dx}{1+\tan\left(\frac{\pi}{4}-x\right)}$ (using property)<br>$I=\int_0^{\pi/4}\frac{dx}{1+\frac{1-\tan x}{1+\tan x}}$ | ½<br>½ |

| | | |
|---|---|---|
| | $I = \frac{1}{2}\int_0^{\pi/4}(1+\tan x)\,dx$ | ½ |
| | $I = \frac{1}{2}\left[x + \log\sec x\right]_0^{\pi/4}$ | 1 |
| | $= \frac{1}{2}\left[\frac{\pi}{4} + \log\sqrt{2}\right] = \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\log 2\right)$ | |
| | $= \frac{\pi}{8} + \frac{1}{4}\log 2$ | ½ |
| **Q8.** | (a) If $\vec{a}$ and $\vec{b}$ are two vectors such that $\left\lvert\vec{a}+\vec{b}\right\rvert = \left\lvert\vec{b}\right\rvert$, then prove that $\left(\vec{a}+2\vec{b}\right)$ is perpendicular to $\vec{a}$.<br>**OR**<br>(b) If $\vec{a}$ and $\vec{b}$ are unit vectors and $\theta$ is the angle between them, then prove that $\sin\frac{\theta}{2} = \frac{1}{2}\left\lvert\vec{a}-\vec{b}\right\rvert$. | **3** |
| **A8.(a)** | $\lvert\vec{a}+\vec{b}\rvert = \lvert\vec{b}\rvert$ | |
| | $\left(\vec{a}+\vec{b}\right)^2 = \left(\vec{b}\right)^2$ | 1 |
| | $\vec{a}^2 + \vec{b}^2 + 2\vec{a}.\vec{b} = \vec{b}^2$ | ½ |
| | $\vec{a}^2 + 2\vec{a}.\vec{b} = 0$ | |
| | $(\vec{a}+2\vec{b}).\vec{a} = 0$ | 1 |
| | $\therefore (\vec{a}+2\vec{b}) \perp \vec{a}$ | ½ |
| | **Or** | |
| **A8.(b)** | Consider | |
| | $\lvert\vec{a}-\vec{b}\rvert^2 = (\vec{a}-\vec{b}).(\vec{a}-\vec{b}) = \lvert\vec{a}\rvert^2 - 2\vec{a}.\vec{b} + \lvert\vec{b}\rvert^2$ | 1 |
| | $= 1 - 2\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta + 1$ | |
| | $= 2 - 2\cos\theta$ | 1 |
| | $= 2\left(2\sin^2\frac{\theta}{2}\right)$ | |
| | $\therefore\ \sin\frac{\theta}{2} = \frac{1}{2}\lvert\vec{a}-\vec{b}\rvert$ | 1 |

| | | |
|---|---|---|
| **Q9.** | Find the equation of the plane passing through the line of intersection of the planes $\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)=10$ and $\vec{r}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)+4=0$ and passing through the point (–2, 3, 1). | **3** |
| **A9.** | Equation of plane through the intersection of given two planes is<br>$\vec{r}\cdot[(\hat{i}+\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}-\hat{k})]-10+4\lambda=0$<br>$\vec{r}\cdot[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}]-10+4\lambda=0$<br>Point (–2, 3, 1) lies on it.<br>$\therefore\ -2-\cancel{4\lambda}+3+9\lambda+1-\lambda-10+\cancel{4\lambda}=0$<br>$8\lambda=8\Rightarrow\lambda=1$<br>$\therefore$ Equation of plane is<br>$\vec{r}\,.(3\hat{i}+4\hat{j})-6=0$ | 1<br><br><br><br>1<br><br>1 |
| **Q10.** | (a) Find :<br>$\int e^{x}.\sin 2x\ dx$<br>**OR**<br>(b) Find :<br>$\int\frac{2x}{(x^2+1)(x^2+2)}dx$ | **3** |
| **A10.(a)** | $I=\int \underset{\text{II}}{e^{x}}\ \underset{\text{I}}{\sin 2x}\,dx$<br>$I=\sin 2x\,e^{x}-\int \underset{\text{I}}{2\cos 2x}\ \underset{\text{II}}{e^{x}}\,dx$<br>$=e^{x}\sin 2x-2[\cos 2x\,e^{x}-\int(-2\sin 2x)\,e^{x}dx]$<br>$I=e^{x}\sin 2x-2\cos 2x\,e^{x}-4I$<br>$5I=e^{x}\sin 2x-2\cos 2xe^{x}$<br>$\therefore I=\frac{1}{5}e^{x}[(\sin 2x-2\cos 2x)]+C$ | 1<br>½<br>½<br>½<br>½ |
| **A10.(b)** | **Or**<br>$\int\frac{2x}{(x^2+1)(x^2+2)}dx$<br>Let $x^2=t$, $2x\,dx=dt$<br>$\therefore\int\frac{2x}{(x^2+1)(x^2+2)}dx=\int\frac{dt}{(t+1)(t+2)}$ | ½ |

| | | |
|---|---|---|
| | Getting $\frac{1}{(t+1)(t+2)} = \frac{1}{t+1} + \frac{-1}{t+2}$ (by Partial Fraction) | 1 |
| | $\int \frac{dt}{(t+1)(t+2)} = \int \frac{dt}{t+1} - \int \frac{dt}{t+2}$ <br> $= \log\|t+1\| - \log\|t+2\| + C$ | 1 |
| | $= \log\left(x^2+1\right) - \log\left(x^2+2\right) + C$ or $\log\left(\frac{x^2+1}{x^2+2}\right) + C$ | ½ |
| | **SECTION—C** <br> **Question Nos. 11 to 14 carry 4 marks each.** | |
| **Q11.** | Three persons A, B and C apply for a job of manager in a private company. Chances of their selection are in the ratio 1 : 2 : 4. The probability that A, B and C can introduce changes to increase the profits of a company are 0.8, 0.5 and 0.3 respectively. If increase in the profit does not take place, find the probability that it is due to the appointment of A. | **4** |
| **A11.** | $E_1$ : $A$ is selected <br> $E_2$ : $B$ is selected <br> $E_3$ : $C$ is selected | ½ |
| | $F$ : increase in profit does not take place | |
| | $P(E_1) = 1/7$, $P(E_2) = 2/7$, $P(E_3) = 4/7$ | 1 |
| | $P(F\|E_1) = 0{\cdot}2$, $P(F\|E_2) = 0{\cdot}5$, $P(F\|E_3) = 0{\cdot}7$ | 1 |
| | $P(E_1\|F) = \frac{P(E_1)\,P(F\|E_1)}{P(E_1)\,P(F\|E_1) + P(E_2)\,P(F\|E_2) + P(E_3)\,P(F\|E_3)}$ <br> $= \frac{\frac{1}{7}\times\frac{2}{10}}{\frac{1}{7}\times\frac{2}{10} + \frac{2}{7}\times\frac{5}{10} + \frac{4}{7}\times\frac{7}{10}}$ | 1 |
| | $= \frac{2}{40} = \frac{1}{20}$ | ½ |
| **Q12.** | Find the area bounded by the curves $y = \|x-1\|$ and $y = 1$, using integration. | **4** |

**A12.**




*(for correct figure)* 1

Area of the bounded region is

$$\int_0^1 \left[1-(1-x)\right]dx+\int_1^2 [1-(x-1)]\,dx$$ 1

$$\int_0^1 x\,dx+\int_1^2 (2-x)\,dx$$

$$=\left.\frac{x^2}{2}\right]_0^1+\left[2x-\frac{x^2}{2}\right]_1^2$$ 1

$$=\frac{1}{2}+\left[4-\frac{4}{2}-2+\frac{1}{2}\right]$$ 1

$$=-1+2=1$$

**Q13.**

(a) Solve the following differential equation :

$$\left(y-\sin^2 x\right)dx+\tan x\,dy=0$$

**OR** **4**

(b) Find the general solution of the differential equation :

$$\left(x^3+y^3\right)dy=x^2y\,dx$$

**A13.(a)**

$$(y-\sin^2 x)\,dx+\tan x\,dy=0$$

$$\frac{dy}{dx}+\frac{y}{\tan x}=\frac{\sin^2 x}{\tan x}$$ 1

$$\frac{dy}{dx}+(\cot x)\,y=\sin x\cos x$$

$$I.F.\ =e^{\int \cot x\,dx}=e^{\log(\sin x)}=\sin x$$ 1

Solution is given by

$$y\sin x=\int \sin^2 x\cos x\,dx$$ ½

| | | |
|---|---|---|
| | $y\sin x = \int t^2 dt \qquad \because \sin x = t, \cos x\, dx = dt$ | ½ |
| | $y\sin x = \dfrac{t^3}{3} + C$ | ½ |
| | $y\sin x = \dfrac{\sin^3 x}{3} + C$ | ½ |
| | **Or** | |
| **A13.(b)** | $\dfrac{dy}{dx} = \dfrac{x^2 y}{x^3 + y^3}$ | ½ |
| | Put $y = vx$, $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ | ½ |
| | $v + x\dfrac{dv}{dx} = \dfrac{v}{1+v^3} \Rightarrow x\dfrac{dv}{dx} = \dfrac{-v^4}{1+v^3}$ | |
| | $\int \dfrac{1+v^3}{v^4} dv = -\int \dfrac{dx}{x}$ | 1 |
| | $\int \dfrac{1}{v^4} dv + \int \dfrac{1}{v} dv = -\log\|x\| + C$ | |
| | $\dfrac{1}{-3v^3} + \log\|v\| = -\log\|x\| + C$ | 1½ |
| | $\dfrac{-x^3}{3y^3} + \log\left\|\dfrac{y}{x}\right\| = -\log\|x\| + C$ or $\dfrac{-x^3}{3y^3} + \log\|y\| = C$ | ½ |

| | | |
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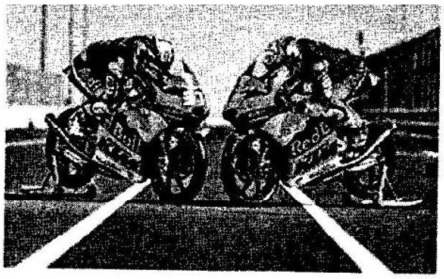
| **Q14.** | Two motorcycles A and B are running at the speed more than the allowed speed on the roads represented by the lines $\vec{r} = \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \left(3\hat{i} + 3\hat{j}\right) + \mu\left(2\hat{i} + \hat{j} + \hat{k}\right)$ respectively.<br><br>Based on the above information, answer the following questions :<br>(a) Find the shortest distance between the given lines.<br>(b) Find the point at which the motorcycles may collide. | **4** |
| **A14.(a)** | $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$ , $\vec{a}_2 = 3\hat{i} + 3\hat{j}$<br>$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$<br>$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix}$<br>$= 3\hat{i} - 3\hat{j} - 3\hat{k}$<br>SD $= \dfrac{\left\|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right\|}{\|\vec{b}_1 \times \vec{b}_2\|}$<br>Now, $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j})(3\hat{i} - 3\hat{j} - 3\hat{k})$<br>$= 9 - 9 = 0$<br>Shortest distance between two lines = 0 | ½<br>1<br>½ |
| **A14.(b)** | Any point on the line $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$ is $\lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$<br>Any point on the line $\vec{r} = 3\hat{i} + 3\hat{j} + \mu(2\hat{i} + \hat{j} + \hat{k})$ is $(2\mu + 3)\hat{i} + (\mu + 3)\hat{j} + \mu\hat{k}$<br>As the lines are intersecting,<br>$\lambda = 2\mu + 3, 2\lambda = \mu + 3$<br>On solving $\mu = -1,\ \lambda = 1$<br>Point of intersection is $\hat{i} + 2\hat{j} - \hat{k}$ or (1, 2, –1) | ½<br>½<br>½<br>½ |