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Senior Secondary School Term II Examination, 2022

Marking Scheme – MATHEMATICS (SUBJECT CODE – 041B)

(PAPER CODE – 65B)

General Instructions: -

1. You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.

2. **"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its' leakage to public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in News Paper/Website etc may invite action under IPC."**

3. Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and marks be awarded to them. In class-XII, while evaluating two competency based questions, please try to understand given answer and even if reply is not from marking scheme but correct competency is enumerated by the candidate, marks should be awarded.**

4. The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.

5. Evaluators will mark(√) wherever answer is correct. For wrong answer 'X' be marked. Evaluators will not put right kind of mark while evaluating which gives an impression that answer is correct and no marks are awarded. **This is most common mistake which evaluators are committing.**

6. If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.

7. If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.

8. If a student has attempted an extra question, answer of the question deserving more marks should be retained and the other answer scored out.

9. No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

10. A full scale of marks __________ (example 0-40 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.

11. Every examiner has to necessarily do evaluation work for full working hours i.e. 8 hours every day and evaluate 30 answer books per day in main subjects and 35 answer books

per day in other subjects (Details are given in Spot Guidelines).This is in view of the reduced syllabus and number of questions in question paper.

12. Ensure that you do not make the following common types of errors committed by the Examiner in the past:-
    - Leaving answer or part thereof unassessed in an answer book.
    - Giving more marks for an answer than assigned to it.
    - Wrong totaling of marks awarded on a reply.
    - Wrong transfer of marks from the inside pages of the answer book to the title page.
    - Wrong question wise totaling on the title page.
    - Wrong totaling of marks of the two columns on the title page.
    - Wrong grand total.
    - Marks in words and figures not tallying.
    - Wrong transfer of marks from the answer book to online award list.
    - Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)
    - Half or a part of answer marked correct and the rest as wrong, but no marks awarded.

13. While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0)Marks.

14. Any unassessed portion, non-carrying over of marks to the title page, or totaling error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, in order to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.

15. The Examiners should acquaint themselves with the guidelines given in the Guidelines for spot Evaluation before starting the actual evaluation.

16. Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.

17. The Board permits candidates to obtain photocopy of the Answer Book on request in an RTI application and also separately as a part of the re-evaluation process on payment of the processing charges.

MARKING SCHEME

Senior Secondary School Examination TERM–II, 2022

MATHEMATICS (Subject Code–041B)

[Paper Code: 65B]

Maximum Marks: 40

***Instructions*:**

1. Instructions for drawing up the MS should be followed carefully.
2. If general instructions have to be given, do so, at the beginning of the page itself.
3. Some subjects will require specific directions for particular type of questions. Give these at the beginning of the concerned question.
4. Do indicate the value points, time required for the questions.
5. Write page number of the topic from where question is asked.

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
1(a)	Find : $\int \frac{1}{\sqrt{12+4x-x^2}}dx$	2
Sol.	$I=\int\frac{1}{\sqrt{12+4x-x^2}}dx$ $=\int\frac{1}{\sqrt{-(x^2-4x+4-16)}}dx=\int\frac{1}{\sqrt{16-(x-2)^2}}dx$ $=\sin^{-1}\frac{x-2}{4}+C$ Or	1 1 2
1(b)	Find : $\int\frac{xe^x}{(x+4)^5}dx$	2
Sol.	$I=\int\frac{xe^x}{(x+4)^5}dx=\int\frac{(x+4)-4}{(x+4)^5}e^x dx$ $=\int\left[\frac{1}{(x+4)^4}-\frac{4}{(x+4)^5}\right]e^x dx$ $=\int[f(x)+f'(x)]\,e^x dx=f(x)\cdot e^x+C \quad \left(\text{where } f(x)=\frac{1}{(x+4)^4}\Rightarrow f'(x)=\frac{-4}{(x+4)^5}\right)$ $=\frac{1}{(x+4)^4}e^x+C$	½ ½ 1 2
2.	Find the general solution of the following differential equation : $(4+y^2)(3+\log x)\,dx+x\,dy=0$	2

Sol.	Given diff. equation can be written as $\frac{(3+\log x)}{x}dx+\frac{1}{4+y^2}dy=0$ Integrating to get $\frac{1}{2}(3+\log x)^2+\frac{1}{2}\tan^{-1}\frac{y}{2}=C_1$ Or, $(3+\log x)^2+\tan^{-1}\frac{y}{2}=C$	1 1 2
3.	If $\lvert\vec{a}\rvert = 3$, $\lvert\vec{b}\rvert = 2\sqrt{3}$ and $\vec{a}.\vec{b} = 6$, then find the value of $\lvert\vec{a}\times\vec{b}\rvert$.	2
Sol.	$6=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\cos\theta \Rightarrow 6=3\cdot2\sqrt{3}\cdot\cos\theta$ $\Rightarrow \cos\theta=\frac{1}{\sqrt{3}}$ $\lvert\vec{a}\times\vec{b}\rvert=\lvert\vec{a}\rvert\lvert\vec{b}\rvert\sin\theta=(3)(2\sqrt{3})\sqrt{1-\frac{1}{3}}$ $=6\sqrt{3}\cdot\frac{\sqrt{2}}{\sqrt{3}}=6\sqrt{2}$	1 1 2
4.	Find the direction cosines of a line whose cartesian equation is given as $3x + 1 = 6y - 2 = 1 - z$.	2
Sol.	Given equation is $3x+1=6y-2=1-z$ $\Rightarrow \frac{x+\frac{1}{3}}{\frac{1}{3}}=\frac{y-\frac{1}{3}}{\frac{1}{6}}=\frac{z-1}{-1}$ $\Rightarrow$ DRs are $\left\langle\frac{1}{3},\frac{1}{6},-1\right\rangle$ or $<2,\ 1,\ -6>$ $\Rightarrow$ DCs are $\frac{2}{\sqrt{41}},\frac{1}{\sqrt{41}},-\frac{6}{\sqrt{41}}$ Or $-\frac{2}{\sqrt{41}},-\frac{1}{\sqrt{41}},\frac{6}{\sqrt{41}}$	1 ½ ½ 2
5	In a toss of three different coins, find the probability of coming up of three heads, if it is known that at least one head comes up.	2

Sol. Let E_1 : getting three heads

E_2 : getting at least one head

$P(E_2)={}^7\!/_8\,;\ P(E_1\cap E_2)={}^1\!/_8$ ½ + ½

$$P(E_1\,|\,E_2)=\frac{P(E_1\cap E_2)}{P(E_2)}=\frac{\frac{1}{8}}{\frac{7}{8}}$$ ½

$$=\frac{1}{7}$$ ½

2

6. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement. 2

Sol. Rotten apples = 2, fresh = 8, total = 10

Let X be the random variable (no. of rotten apples)

X can take values 0, 1, 2

Probability Distribution is

X	0	1	2
$P(X)$	$\frac{{}^8C_2}{{}^{10}C_2}=\frac{28}{45}$	$\frac{{}^8C_1\cdot{}^2C_1}{{}^{10}C_2}=\frac{16}{45}$	$\frac{{}^2C_2}{{}^{10}C_2}=\frac{1}{45}$

½

1½

2

7. Evaluate : $\int_0^{\pi/3} |\cos 3x|\,dx$ 3

Sol.

$$I=\int_0^{\pi/3}|\cos 3x|\,dx$$

$$=\int_0^{\pi/6}\cos 3x dx-\int_{\pi/6}^{\pi/3}\cos 3x\,dx$$ 1½

$$=\left.\frac{\sin 3x}{3}\right]_0^{\pi/6}-\left.\frac{\sin 3x}{3}\right]_{\pi/6}^{\pi/3}$$ ½

$$=\frac{1}{3}[1-0]-\frac{1}{3}[0-1]=\frac{2}{3}$$ 1

3

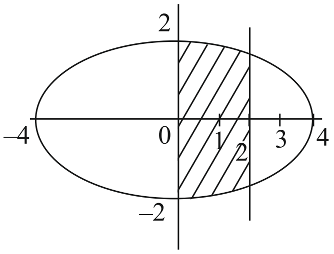
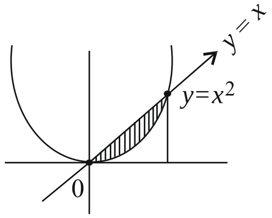
8.(a) Find the general solution of the following differential equation :

$$2x\,e^{y/x}\,dy+(x-2y\,e^{y/x})\,dx=0$$

3

Sol.	Given diff. equation is $\frac{dy}{dx}=\frac{2ye^{y/x}-x}{2xe^{y/x}}$ Put, $\frac{y}{x}=v \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ $\therefore \quad v+x\frac{dv}{dx}=\frac{2ve^{v}-1}{2e^{v}}$ $\Rightarrow x\frac{dv}{dx}=\frac{2ve^{v}-1}{2e^{v}}-v=-\frac{1}{2e^{v}}$ $\Rightarrow 2e^{v}dv=-\frac{dx}{x}$ Integrating to get, $2e^{v}=-\log\|x\|+C$ $\Rightarrow 2e^{y/x}+\log\|x\|=C$	½ ½ ½ ½ 1
	Or	3
8.(b)	Find the particular solution of the differential equation $(2x^2+y)\cdot\frac{dx}{dy}=x$; given that $y=2$ when $x=1$.	3
Sol.	Given diff. equation can be written as $x\frac{dy}{dx}-y=2x^2 \quad or \quad \frac{dy}{dx}-\frac{1}{x}y=2x$ IF $= e^{\int -\frac{1}{x}dx}=e^{-\log x}=e^{\log\frac{1}{x}}=\frac{1}{x}$ $\therefore$ Solution is $y\cdot\frac{1}{x}=\int 2x\cdot\frac{1}{x}dx=2x+C$ $\Rightarrow y=2x^2+Cx$ when $x=1,\ y=2 \Rightarrow 2=2+C \Rightarrow C=0$ $\Rightarrow$ Particular Solution is $y=2x^2$	1 ½ ½ ½ ½
		3
9.	Using vectors, find the area of the triangle with vertices A(–1, 0, – 2), B(0, 2, 1) and C(–1, 4, 1).	3
Sol.	Taking any two sides (say AB and AC) $\overrightarrow{AB}=\hat{i}+2\hat{j}+3\hat{k}$ and $\overrightarrow{AC}=0\hat{i}+4\hat{j}+3\hat{k}$ $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 0 & 4 & 3\end{vmatrix}=-6\hat{i}-3\hat{j}+4\hat{k}$	1 1

	Area of $\Delta = \frac{1}{2}\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert = \frac{1}{2}\sqrt{36+9+16} = \frac{1}{2}\sqrt{61}$	1 3
10.(a)	Find the shortest distance between the lines $\vec{r} = (\lambda + 1)\,\hat{i} + (\lambda + 4)\,\hat{j} - (\lambda - 3)\,\hat{k}$, and $\vec{r} = (3 - \mu)\,\hat{i} + (2\mu + 2)\,\hat{j} + (\mu + 6)\,\hat{k}$.	3
Sol.	Lines are $\vec{r} = \hat{i} + 4\hat{j} + 3\hat{k} + \lambda(\hat{i} + \hat{j} - \hat{k})$ and $\vec{r} = 3\hat{i} + 2\hat{j} + 6\hat{k} + \mu(-\hat{i} + 2\hat{j} + \hat{k})$ $\vec{a}_2 - \vec{a}_1 = 2\hat{i} - 2\hat{j} + 3\hat{k}$ $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ -1 & 2 & 1 \end{vmatrix} = 3\hat{i} + 0\hat{j} + 3\hat{k}$ $\text{SD} = \frac{\lvert(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\rvert}{\lvert \vec{b}_1 \times \vec{b}_2 \rvert} = \frac{6+9}{\sqrt{18}} = \frac{15}{3\sqrt{2}} = \frac{5}{\sqrt{2}}$ Or	½ ½ 1 1 3
10.(b)	Find the distance of the point P(4, 3, 2) from the plane determined by the points A(−1, 6, − 5), B(− 5, − 2, 3) and C(2, 4, − 5).	3
Sol.	Equation of plane is $\begin{vmatrix} x+1 & y-6 & z+5 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0$ $\Rightarrow 16(x+1) + 24(y-6) + 32(z+5) = 0$ $\Rightarrow 2x + 3y + 4z + 4 = 0$ Required Distance $= \frac{\lvert 8+9+8+4 \rvert}{\sqrt{4+9+16}} = \sqrt{29}$	1 1 1 3
11.	Find : $\int \frac{x^2 + x + 1}{(x+1)(x^2+4)}\,dx$	4
Sol.	$I = \int \frac{x^2 + x + 1}{(x+1)(x^2+4)}\,dx$ $\frac{x^2 + x + 1}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx + C}{x^2+4} = \frac{\frac{1}{5}}{x+1} + \frac{\frac{4}{5}x + \frac{1}{5}}{x^2+4}$	½ + 1½

	$\Rightarrow I = \int \frac{\frac{1}{5}}{x+1} dx + \frac{2}{5}\int \frac{2x}{x^2+4} dx + \frac{1}{5}\int \frac{dx}{x^2+4}$ $= \frac{1}{5}\log\lvert x+1\rvert + \frac{2}{5}\log\lvert x^2+4\rvert + \frac{1}{10}\tan^{-1}\frac{x}{2} + C$	½ 1½
		4
12.(a)	Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.	4
Sol.	$A = 2\int_0^2 y\,dx$ $= 2\cdot\frac{1}{2}\cdot\int_0^2 \sqrt{16-x^2}\,dx$ $= \left[\frac{x}{2}\sqrt{16-x^2} + 8\sin^{-1}\frac{x}{4}\right]_0^2$ $= 2\sqrt{3} + 8\cdot\frac{\pi}{6} = \left(2\sqrt{3} + \frac{4\pi}{3}\right)$  	1 2 1
	Or	4
12.(b)	Find the area of the region $\{(x, y) : x^2 \le y \le x\}$, using integration.	4
Sol.	Getting x = 0, x = 1 for the point of intersection $A = \int_0^1 x\,dx - \int_0^1 x^2 dx$ $= \left.\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$ $= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$  	1 1 1 1
		4
13.	Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.	4
Sol.	Equation of line through (4, –3, –4) and (3, –2, 2) is $\frac{x-4}{-1} = \frac{y+3}{1} = \frac{z+4}{6}$ Any point on the line is $(-\lambda+4,\ \lambda-3,\ 6\lambda-4)$ If the point lies on the plane, then $2(-\lambda+4) + (\lambda-3) + 6\lambda - 4 = 6$ $\Rightarrow 5\lambda = 5 \Rightarrow \lambda = 1$	1 1 1

	∴ Point is $(3, -2, 2)$	1
		4
14.	A laboratory blood test is 98% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0·4% of the healthy person tested. From a large population, it is given that 0·2% of the population actually has the disease. Based on the above, answer the following questions : (a) One person, from the population, is taken at random and given the test. Find the probability of his getting a positive test result. (b) What is the probability that the person actually has the disease, given that his test result is positive ?	4
Sol.	Let $\left.\begin{array}{l} E_1 : \textit{person has the disease} \\ E_2 : \textit{person does not have the disease} \\ A : \textit{getting a } +ve \textit{ test report} \end{array}\right\}$	1
	$P(E_1) = \frac{2}{1000}, \quad P(E_2) = \frac{998}{1000}$	½
	$P(A \mid E_1) = \frac{98}{100}, \quad P(A \mid E_2) = \frac{4}{1000}$	½
(a)	$P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$ $= \frac{2}{1000} \times \frac{98}{100} + \frac{998}{1000} \times \frac{4}{1000} = \frac{5952}{1000000}$	½ + ½
(b)	$P(E_1 \mid A) = \frac{P(E_1)P(A \mid E_1)}{\Sigma P(E_i)\, P(A \mid E_i)}$ $= \frac{\frac{2}{1000} \times \frac{980}{1000}}{\frac{5952}{1000000}} = \frac{1960}{5952} = \frac{245}{744}$	½ + ½
		4